

COMPARISON BETWEEN THE METHOD OF THE MAXIMUM LIKELIHOOD AND BAYESIAN ESTIMATING THE PARAMETERS OF THE GENERALIZED EXTREME VALUE DISTRIBUTION OF THE FIRST TYPE WITH AN APPLICATION TO TEMPERATURES IN BASRA GOVERNORATE

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Abstract: A comparison was made between the method of maximum Likelihood and the method of Bayes in estimating the parameters of the (GEV) (Generalized Extreme Value distribution), by using Newton Raphson to find nonlinear equations which producer from the estimation and the results showed that the estimators of the non-informative Bayes method are better in estimation than the method of maximum Likelihood (ML), where the Bayes estimators (MSE) are smaller than the estimators (MLE) with sample sizes (25,50,100), and at the sample size (500) the estimators were similar to the assumption values of the parameters, real data were used on the applied side and some statistical indicators were given that showed that random temperature data suffer from extremism only and do not suffer from anomalies, skewness or, kurtosis which makes the distribution of the (GEVD) suitable for representing temperature data.

INTRODUCTION

The Extreme Value distribution describes the random behavior of variables whose observations include very large and small values such as wind speed, temperature, earthquake, flood, rain, and other natural phenomena [1, 3, 5]. The (Extreme Value Distribution) has been proposed based on the Extreme value theory, which is the theory that is used if observations include small or very large values, and although the frequency of these values is relatively small, but they affect the shape of the distribution and the size of the tail [2, 10].

von Bortkiewicz [11] developed the methodology of Extreme Value when he examined the distribution of the extent to which samples accept the normal distribution and reached to clarify the concept of maximum value. The researcher Gumbel was the first to draw the attention of engineers and statisticians to the probabilistic applications of the Extreme Value theory, and among the problems that were addressed with Extreme value distributions for the first time are the problems that Related to meteorological phenomena in the United States of America, annual flood incidents and maximum rainfall and that was in 1940 [3].

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The Research Problem: The Extreme Value are large or very small values so that they move away from the prevailing values in the sample, which are few, but they are important in clarifying the behavior of the random variable that characterizes the phenomenon and therefore cannot be dispensed with, which is one of the problems facing the researcher.

Research Objective: The aim of this study is to compare the generalized Extreme Value distribution (GEV) estimators that were found by the ML method and the Non-Informative Bayesian Method and to determine which is better in representing temperature data.

Generalized Extreme Value Distribution (GEVD)

The first to propose the generalized Extreme Value distribution is the researcher Jenkinson in general [5], and this distribution has two formulas for the Cumulative function depending on the presence of the shape parameter and the domain of the random variable The Cumulative function of the generalized Extreme Value distribution is written as follows

$$F(x) = \begin{cases} e^{-\left(1+\varepsilon\left(\frac{x-\alpha}{\beta}\right)\right)^{\frac{1}{\varepsilon}}} & -\infty < x \leq \left(\frac{x-\alpha}{\beta}\right) \text{ for } \varepsilon < 0 \\ \left(\frac{x-\alpha}{\beta}\right) \leq x < \infty \text{ for } \varepsilon > 0 \\ e^{-e^{-\left(\frac{x-\alpha}{\beta}\right)}} & -\infty < x < \infty \text{ for } \varepsilon = 0 \end{cases} \quad (1)$$

As the shape parameter ε , α is a location parameter, β a scale parameter.

$$-\infty \leq \varepsilon; \alpha \leq \infty \beta \geq 0$$

In the case of a ($\varepsilon > 0$) Frechet Distribution or a generalized Extreme Value distribution of the second type, either if ($\varepsilon < 0$) a Weibull distribution or a generalized Extreme Value distribution of the third type is obtained, and in the case of a sim obtained ($\varepsilon = 0$) Gumbel distribution dis. Or generalized Extreme Value distribution of the first type The probability density function of the generalized Extreme Value distribution is [2, 3, 5-7]:

$$f(x) = \begin{cases} \frac{1}{\beta} \left(1 + \varepsilon \left(\frac{x-\alpha}{\beta}\right)\right) e^{-\left(1+\varepsilon\left(\frac{x-\alpha}{\beta}\right)\right)^{\frac{1}{\varepsilon}}} & -\infty < x \leq \left(\frac{x-\alpha}{\beta}\right) \text{ for } \varepsilon < 0 \\ \left(\frac{x-\alpha}{\beta}\right) \leq x < \infty \text{ for } \varepsilon > 0 \\ \frac{1}{\beta} e^{-\left(\frac{x-\alpha}{\beta}\right)} e^{-e^{-\left(\frac{x-\alpha}{\beta}\right)}} & -\infty < x < \infty \text{ for } \varepsilon = 0 \end{cases} \quad (2)$$

The diagram shows the curve of the probability density function and the associative function for the distribution of Extreme value of the first type of assumption values

of the distribution parameters

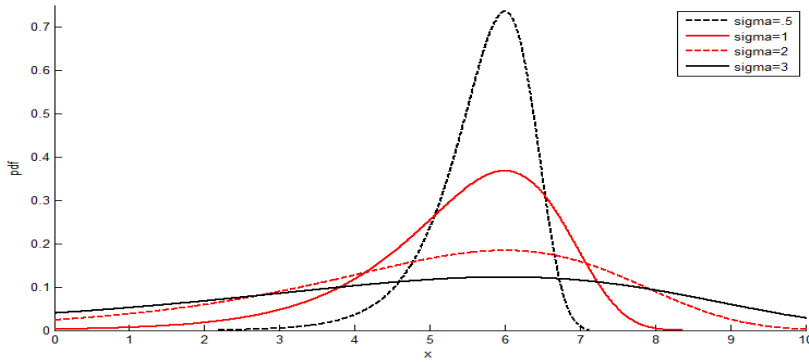


Figure 1: Curve of the Probability Density Function for the Distribution of the Extreme Value of the First Type With a Assumption Values For the Location Parameter ($\alpha = 6$) and Assumption Values for the Scale Parameter ($\beta = 0.5; 1; 2; 3$).

The figure shows how the probability density function curve affects at the Extreme Values

Estimation Methods

The parameters of the distribution of (GEV) of the first type were estimated using the method of Maximum Likelihood and the non-informational Bayes, which does not depend on Fisher information.

Maximum Likelihood Method [1]

The Maximum likelihood method of the most used traditional methods of estimation, developed by the scientist (Fisher) in 1922, where the estimators of this method are characterized by the properties of a good estimator such as Unbiased, Consistency and Invariant, according to this method, the probability function is found for sample observations and then the estimators that make the Maximum Likelihood function at its maximum limit are found.

To find the estimators of the of the (ML) of (GEV) of the first type, of the (pdf), we get the following [1]:

$$L(\alpha, \beta, \chi_i) = \frac{1}{\beta^n} e^{-\sum e^{-\left(\frac{\chi_i - \alpha}{\beta}\right)}} \cdot e^{-\sum_{i=1}^n \left(\frac{\chi_i - \alpha}{\beta}\right)} \quad (3)$$

find the natural logarithm of equation (1) produces

$$\mathcal{L}(\alpha, \beta, \chi_i) = -n \ln(\beta) - \sum_{i=1}^n e^{-\frac{\chi_i - \alpha}{\beta}} - \sum_{i=1}^n \left(\frac{\chi_i - \alpha}{\beta}\right) \quad (4)$$

According to the method of maximum likelihood, the partial derivative of equation (4) will be found for the parameters to find the maximum limit of the logarithm of the probability function. Equation ($\alpha; \beta$) (4) shows the first partial derivative of equation (4) with respect to the location parameter (α)

$$\frac{\partial \mathcal{L}}{\partial \hat{\alpha}} = \frac{n}{\hat{\beta}} - \frac{1}{\hat{\beta}} \sum_{i=1}^n e^{-\frac{\chi_i - \hat{\alpha}}{\hat{\beta}}} \quad (5)$$

find the partial derivative of equation (2) for the scale parameter (β)

$$\frac{\partial \mathcal{L}}{\partial \hat{\beta}} = -\frac{n}{\hat{\beta}} - \frac{1}{\hat{\beta}^2} \sum_{i=1}^n (\chi_i - \hat{\alpha}) e^{-\frac{\chi_i - \hat{\alpha}}{\hat{\beta}}} - \frac{n\hat{\alpha}}{\hat{\beta}^2} + \sum_{i=1}^n \frac{\chi_i}{\hat{\beta}} \quad (5)$$

Equating equations (5) and (6) with zero results in the following

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \hat{\alpha}} &= \frac{n}{\hat{\beta}} - \frac{1}{\hat{\beta}} \sum_{i=1}^n e^{-\frac{\chi_i - \hat{\alpha}}{\hat{\beta}}} = 0 \\ -\frac{n}{\hat{\beta}} - \frac{1}{\hat{\beta}^2} \sum_{i=1}^n (\chi_i - \hat{\alpha}) e^{-\frac{\chi_i - \hat{\alpha}}{\hat{\beta}}} - \frac{n\hat{\alpha}}{\hat{\beta}^2} + \sum_{i=1}^n \frac{\chi_i}{\hat{\beta}} &= 0 \end{aligned}$$

The system of equations (5) and (6) is a system of nonlinear equations that cannot be solved by classic mathematical methods and one of the numerical methods can be used to find estimates of the (ML) function. Newton-Raphson's method is one of the most important and widely used methods to solve a system of nonlinear equations resulting from the method of Maximum likelihood.

Bayes' Method for Estimating Parameters of Generalized Extreme Value Distribution [6, 8-10]

The parameters of the generalized Extreme Value distribution will be estimated using the non-informative Bayes method by applying the Jeffery method, which does not depend on Fisher Information, we assume according to the Jeffery method that the prior non-informational function of the location parameter is (α)

$$\pi_1(\alpha) = \text{constant} \quad -\infty \leq \alpha \leq \infty \quad (7)$$

We will also assume that the prior non-informational, marginal probability function of the scale parameter (β) is

$$\pi_2(\beta) \propto \frac{1}{\beta} \quad 0 \leq \beta \leq \infty \quad (8)$$

The common prior joint probability function of the location and Scale parameter is

$$\pi(\alpha ; \beta) \propto \frac{1}{\beta} \quad (9)$$

From equation (1) and equation (7) the common distribution of the maximum probability function and the Joint prior non-informational probability function are obtained as follows:

$$f(\chi, \alpha, \beta) = \frac{1}{\beta^{n+1}} e^{-\Sigma\left(\frac{\chi_i - \alpha}{\beta}\right)} e^{-e^{-\Sigma\left(\frac{\chi_i - \alpha}{\beta}\right)}} \quad (10)$$

Using Bayes' law is obtained (posterior Density Distribution):

$$h(\alpha, \beta | \chi) = \frac{\frac{1}{\beta^{n+1}} e^{-\Sigma\left(\frac{\chi_i - \alpha}{\beta}\right)} e^{-e^{-\Sigma\left(\frac{\chi_i - \alpha}{\beta}\right)}}}{\int \int \frac{1}{\beta^{n+1}} e^{-\Sigma\left(\frac{\chi_i - \alpha}{\beta}\right)} e^{-e^{-\Sigma\left(\frac{\chi_i - \alpha}{\beta}\right)}} d\alpha d\beta} \quad (11)$$

Assuming a quadratic loss function:

$$u(\underline{\theta}) = L(\hat{\underline{\theta}} - \underline{\theta})^2 \quad (12)$$

Whereas: represents the estimated feature vector: represents the feature vector $\hat{\underline{\theta}}(\alpha; \beta)$

The Bayes Estimator is the estimator that makes the prediction of the loss function at its minimum limit, i.e.

$$\text{Min} E \left(L(\hat{\underline{\theta}} - \underline{\theta})^2 \right) = \text{Min} \left(\frac{\int \int (\hat{\underline{\theta}} - \underline{\theta})^2 \frac{1}{\beta^{n+1}} e^{-\Sigma\left(\frac{\chi_i - \alpha}{\beta}\right)} e^{-e^{-\Sigma\left(\frac{\chi_i - \alpha}{\beta}\right)}} d\alpha d\beta}{\int \int \frac{1}{\beta^{n+1}} e^{-\Sigma\left(\frac{\chi_i - \alpha}{\beta}\right)} e^{-e^{-\Sigma\left(\frac{\chi_i - \alpha}{\beta}\right)}} d\alpha d\beta} \right) \quad (13)$$

Integration (10) cannot be solved by the classic mathematical method, so Lindley Approximation will be used to obtain the Bayes-method estimators for generalized extreme value distribution parameters.

Lindley Approximation

In (1980), Lindley [4] developed a formula known as Lindley Approximation to find Bayes' estimators by finding the approximate solution for the following integration:

$$E_h \left(L(\hat{\underline{\theta}} - \underline{\theta}) \right) = \frac{\int u(\underline{\theta}) e^{L(h(\underline{\theta})) + p(\underline{\theta})} \partial \underline{\theta}}{\int e^{L(h(\underline{\theta})) + p(\underline{\theta})} \partial \underline{\theta}}$$

Whereas,

$u(\underline{\theta})$: Any function of the parameter or vector of features (loss function) $(\theta)(\underline{\theta})$

$L(h(\underline{\theta}))$: Logarithm of the probability function for sample data

$p(\underline{\theta})$: logarithm of the preceding distribution function of the parameter (parameter vector)

The above integration is solved using the Landley approximation, as Landley suggested the approximation below

$$E(u(\underline{\theta})) = u(\hat{\underline{\theta}}) + \frac{1}{2} \sum_{i=1}^M \sum_{j=1}^M (u_{ij} - 2u_i \rho_j) \sigma_{ij} + \frac{1}{2} \sum_{i=1}^M \sum_{j=1}^M \sum_{k=1}^M \sum_{L=1}^M L_{ijkl} u_i \sigma_{ij} \sigma_{kL}$$

Whereas,

M: represents the number of parameters to be estimated ($M = 2$)

$u(\hat{\theta})$: Weighting Method Estimator (MLS)

When assuming

$$\begin{aligned}
 u(\hat{\beta}) &= \hat{\beta}_{mle} \\
 u_{\alpha} &= \frac{\partial u}{\partial \alpha} = 1; u_{\beta} = \frac{\partial u}{\partial \beta} = 1; \rho_1 = \frac{\partial p(\alpha; \beta)}{\partial \alpha} = 0; \rho_2 = \frac{\partial(\alpha; \beta)}{\partial \beta} = -\frac{1}{\beta} \\
 u_{\alpha\alpha} &= \frac{\partial^2 u}{\partial \alpha^2} = u_{\alpha\beta} = \frac{\partial^2 u}{\partial \alpha \partial \beta} = u_{\beta\beta} = \frac{\partial^2 u}{\partial \beta^2} = 0; \\
 \sigma_{\alpha\alpha} &= -\left(\frac{\partial^2 \mathcal{L}}{\partial \alpha^2}\right)^{-1} = \frac{\beta^2}{\sum e^{-\left(\frac{\chi_i - \alpha}{\beta}\right)}}; \sigma_{\alpha\beta} = -\left(\frac{\partial^2 \mathcal{L}}{\partial \alpha \partial \beta}\right)^{-1} = \frac{\beta^2}{\sum e^{-\left(\frac{\chi_i - \alpha}{\beta}\right)}(\beta - \chi_i + \alpha)} \\
 \sigma_{\beta\beta} &= -\left(\frac{\partial^2 \mathcal{L}}{\partial \beta^2}\right)^{-1} = \frac{\beta^4}{\sum e^{-\left(\frac{\chi_i - \alpha}{\beta}\right)}(2\beta - \chi_i + \alpha)(\alpha - \chi_i) - n\beta(\beta + 2\alpha) + 2\beta \sum \chi_i} \\
 L_{\alpha\alpha\alpha} &= \frac{\partial^3 \mathcal{L}}{\partial \alpha^3} = \frac{1}{\beta^3} \sum e^{-\left(\frac{\chi_i - \alpha}{\beta}\right)} \\
 L_{\alpha\alpha\beta} &= \frac{\partial^3 \mathcal{L}}{\partial \alpha^2 \partial \beta} = \frac{1}{\beta^4} \sum e^{-\left(\frac{\chi_i - \alpha}{\beta}\right)}(2\beta - \chi_i + \alpha) = L_{\alpha\beta\alpha} = L_{\beta\alpha\alpha} \\
 L_{\alpha\beta\beta} &= \frac{\partial^3 \mathcal{L}}{\partial \alpha \partial \beta^2} = \frac{1}{\beta^5} \sum e^{-\left(\frac{\chi_i - \alpha}{\beta}\right)}(2\beta^2 - 4\beta\chi_i + 4\alpha\beta + \chi_i^2 - 2\chi_i\alpha + \alpha^2) - 2n\beta^2 \\
 L_{\alpha\beta\beta} &= L_{\beta\alpha\beta} = L_{\beta\beta\alpha} \\
 L_{\beta\beta\beta} &= \frac{\partial^3 \mathcal{L}}{\partial \beta^3} = \frac{1}{\beta^6} \sum (\alpha - \chi_i)e^{-\left(\frac{\chi_i - \alpha}{\beta}\right)}((6\beta - 6\chi_i + 6\alpha)\beta - \chi_i^2 - 2\chi_i\alpha + \alpha^2) + 6n\alpha\beta^2 + 6\beta^2 \sum \chi_i \\
 \alpha_{bay} &= \alpha_{mle} - \rho_2\sigma_{22} + \frac{1}{2}(\sigma_{\alpha\alpha} + \sigma_{\alpha\beta})L_{\beta\alpha\alpha}\sigma_{\alpha\beta} + \frac{1}{2}[(\sigma_{21} + \sigma_{22}) + (\sigma_{11} + \sigma_{12})]L_{\beta\alpha\beta} + \frac{1}{2}(\sigma_{\beta\alpha} + \sigma_{\beta\beta})L_{\beta\beta\beta} \\
 \beta_{bay} &= \beta_{mle} - \rho_2\sigma_{22} + \frac{1}{2}(\sigma_{\alpha\alpha} + \sigma_{\alpha\beta})L_{\beta\alpha\alpha}\sigma_{\alpha\beta} + \frac{1}{2}[(\sigma_{21} + \sigma_{22}) + (\sigma_{11} + \sigma_{12})]L_{\beta\alpha\beta} + \frac{1}{2}(\sigma_{\beta\alpha} + \sigma_{\beta\beta})L_{\beta\beta\beta}
 \end{aligned}$$

Simulation

The Monte Carlo simulation method was used for the purpose of comparing the method of the (ML) and the non-informative Bayes method in estimating the parameters of the (GEV) distribution of the first type, and the inverse method was used to obtain random variables that follow the distribution of Extreme Values of the first type and according to the following formula

$$x = \alpha - \ln(-\ln(u))\beta$$

Table 1: Hypothesis Parameters.

The sample sizes were assumed as the parameter values shown in the following table ($n = 15; 25; 50; 100; 500$) Table (1):25				
12	4	2	α	
10	4	1	0.2	β

The simulation process was repeated 1000 to ensure the homogeneity of its results, and the criterion of Mean Squares Error was used to compare the estimation methods for each model.

$Mse = \sum_{l=1}^L \frac{(\theta - \hat{\theta})}{L}$; As the number of repetitions $L(L = 1000)$.

The following results have been obtained:

Table 2: Results of the Estimators of the (ML) Method, the Bayes Method and the Value of mse ($\alpha = 2; \beta = 0.2$).

Best		Bayes Method				MLE Method				N
β	α	$mse(\beta)$	$mse(\alpha)$	$\hat{\beta}$	$\hat{\alpha}$	$mse(\beta)$	$mse(\alpha)$	$\hat{\beta}$	$\hat{\alpha}$	
Bay	Bay	0.00001	0.00001	0.19533	1.99533	0.00103	0.00158	0.19381		25
Bay	Bay	0.00000	0.00000	0.19798	1.99798	0.00050	0.00095	0.19699	2.00203	50
Bay	Bay	0.00001	0.00001	0.19603	1.99603	0.00024	0.00042	0.19752	2.00068	100
Bay	Bay	0.00000	0.00000	0.19987	1.99987	0.00005	0.00008	0.19988	2.00014	500

Table 2: Results of the Estimators of the Method of the (ML) and the Method of Bayes and the Value of When $mse(\alpha = 4; \beta = 1)$.

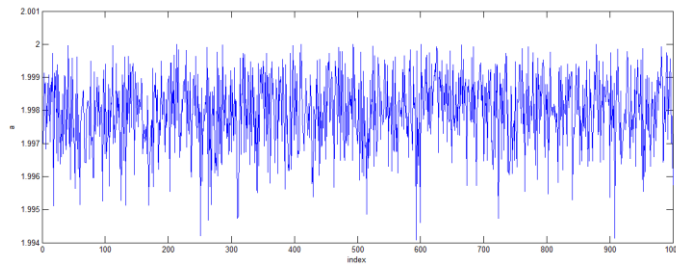
Best		Bay Method				MLE Method				N
		$mse(\beta)$	$mse(\alpha)$	$\hat{\beta}$	$\hat{\alpha}$	$mse(\beta)$	$mse(\alpha)$	$\hat{\beta}$	$\hat{\alpha}$	
Bay	Bay	0.000342	0.000342	0.977182	3.977182	0.026410	0.063400	0.965590	4.013700	25
Bay	Bay	0.000056	0.000056	0.990129	3.990129	0.012438	0.021974	0.986152	4.009157	50
Bay	Bay	0.000000	0.000000	0.999355	3.999355	0.005869	0.011193	0.996419	4.007810	100
Bay	Bay	0.000000	0.000000	0.999355	3.999355	0.001249	0.002131	0.998560	4.000964	500

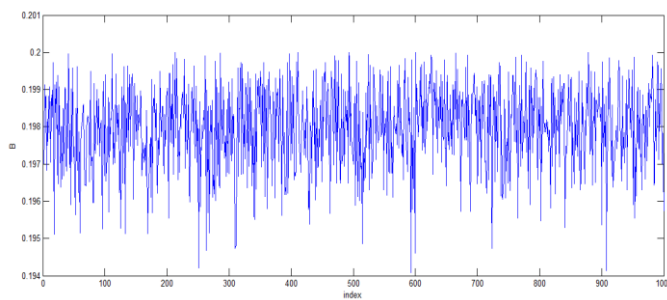
Table 3: Results of the estimators of the Maximum Likelihood Method, the Bayes Method and the Value of mse with ($\alpha = 12; \beta = 4$).

Best		Bay Method				MLE Method				N
β	A	$mse(\beta)$	$mse(\alpha)$	$\hat{\beta}$	$\hat{\alpha}$	$mse(\beta)$	$mse(\alpha)$	$\hat{\beta}$	$\hat{\alpha}$	
Bay	Bay	0.018453	0.018453	3.91148	11.91148	0.002358	0.000205	5.535561	11.54684	25
Bay	Bay	0.000872	0.000872	3.961227	11.96123	0.002358	0.000205	3.91081	12.00175	50
Bay	Bay	0.000114	0.000114	3.984241	11.98424	0.098179	0.179144	3.963681	12.02146	100
Bay	Bay	2.38E-06	2.38E-06	3.997432	11.99743	0.02208	0.034792	3.990498	12.00339	500

Table 4: Results of the Estimators of The Method of the ML, the Bayes Method and the Value of mse With ($\alpha = 25; \beta = 10$).

Best		Bay Method				MLE Method				N
B	A	$mse(\beta)$	$mse(\alpha)$	$\hat{\beta}$	$\hat{\alpha}$	$mse(\beta)$	$mse(\alpha)$	$\hat{\beta}$	$\hat{\alpha}$	
Bay	Bay	0.069086	0.069086	9.773959	24.773960	0.379662	0.103000	10.616170	25.320940	25
Bay	Bay	0.006038	0.006038	9.900669	24.900670	1.184395	2.089672	9.831035	25.103380	50
Bay	Bay	0.001962	0.001962	9.960215	24.960220	0.659504	1.077173	9.912976	25.070470	100
Bay	Bay	0.000016	0.000016	9.993589	24.993590	0.112818	0.206338	9.984255	24.999260	500





It is clear from Tables (1), (2), (3), (4) that each of $(\alpha), (\beta)$ with different assumption values and sample sizes ($n=25, 50, 100$) estimated in two ways (Informative Bayes & MLE), that the estimate by the Bayes method is better than the estimate by the (MLE), because the (MSE) for Bayesian estimators is smaller than for estimators by the ML method, When the sample size increased ($n=500$), the Bayesian estimators $(\alpha), (\beta)$ were similar to the default values of these parameters, as MSE became equal to zero.

Applied Side

The applied aspect included finding the temperature distribution in Basra Governorate, as the data included the temperatures recorded for the period (2022-2023) and by two readings for each day, the Easy Fit program was used to find the distribution of the recorded data, and it was found that the random behavior of the data according to the distribution of Extreme values of the first type, and some descriptive statistics were calculated as shown in the table below

Kurtosis	Variance	Skewness	Min	Max	Median	Standard Deviation	Mode	Mean
-0.915	149.8	0.142	1	53	28	10.16	31	28.186

From the results of the measures of the central tendency above (Mean, Mode, Median,), which are very close and are respectively (28,31, 28.186), it is clear that most of the values of these data are distributed naturally , so to maintain the convexity of the distribution curve, and that these data are independent and similar on both sides of the distribution, but that the Extreme Values affect the tail of the distribution, which is heavier than the one sides, and these values, although they are a few, but cannot be deleted as they describe the phenomenon (temperatures in Basra). This confirms that the real data suffer from extremes, which is evident from the difference between the highest and lowest value , the(Range = 52), but it is devoid of outliers that do not belong to the phenomenon, as well as it does not suffer from , skewness and deviation, this makes the shape of the distribution curve similar to the normal distribution with a longer tail on the one sides due to the Extreme value , and therefore it is proportional to the Gumbel distribution of Extreme Values.

Estimating the Parameters of The Distribution

Method		Parameter
Bayes	Maximum Likelihood	
22.1603	28.1626	α
10.9783	12.2336	β

Choosing the Best Model 2

Akaike (ACI), BIC, Bayesian Information Criterion, and Hannan Quinn Information Criterion(HQIC) were used to find the best model to represent the random behavior of temperature data in Basra Governorate according to equations.:

$$AIC = -2\mathcal{L}(\hat{\theta} \mid x) + 2q \ ; \ BIC = -2\mathcal{L}(\hat{\theta} \mid x) + q\log(n)$$

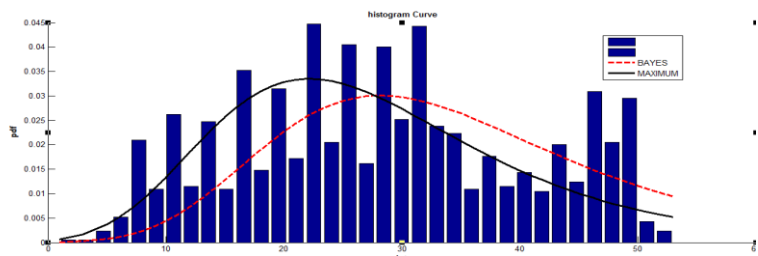
$\mathcal{L}(\hat{\theta} \mid x)$: The natural logarithm of the greatest weighting function

q : Number of parameters; n : the Observations

Table 5: Shows the Values of the Differentiation Criteria Between the Estimation Methods for Temperature Data in Basra Governorate.

Criterion	Maximum Likelihood	Bayes Method
AIC	5233.689	7089.736
BIC	5223.174	7079.224
H-Q	5229.759	7085.809

From Table (5), it is clear from the three differential criteria (AIC, BIC, H-Q) used to find the best method for estimating the parameters of the (GEVD) distribution that the Bayesian method was the best in estimation, as the value of these criteria at the Bayesian method is greater than its value at (MLE).



REFERENCES

[1] I. F. Alves, C. Neves, *Extreme Value Distributions*, in: M. Lovric (Ed.) International Encyclopedia of Statistical Science, Springer, 2011, 493-496.

[2] A. N. Hussein, *Finding the Efficient Algorithm for Estimating the Parameters of the Mixed Weibull Distribution: Application to Wind Speed in Iraq*, University theses, University of Baghdad - College of Administration and Economics - Department of Statistics, 2015.

[3] S. Kotz, S. Nadarajah, *Extreme Value Distributions: Theory and Applications*, World Scientific, 2000.

[4] D. V. Lindley, *Approximate Bayesian Methods*, Trab. Estad. Invest. Oper. **31** (1980), 223-245.

[5] F. Nascimento, M. Bourguignon, J. Leão, *Extended Generalized Extreme Value Distribution with Applications in Environmental Data*, Hacet. J. Math. Stat. **45** (2016), 1847-1864.

[6] B. Ning, P. Bloomfield, *Bayesian Inference for Generalized Extreme Value Distribution With Gaussian Copula Dependence*, arXiv e-prints (2017), arXiv: 1703.00968.

[7] P. Pawlas, D. Szydal, *Characterizations of the Inverse Weibull Distribution*

and Generalized Extreme Value Distributions by Moments of Kth Record Values, Appl. Math. **27** (2000), 197-202.

- [8] S. B. Provost, A. Saboor, G. M. Cordeiro, M. Mansoor, *On the q -Generalized Extreme Value Distribution*, Revstats - Statistical Journal **16** (2018), 45-70.
- [9] I. Sadok, M. Zribi, A. Masmoudi, *Non-Informative Bayesian Estimation in Dispersion Models*, Hacet. J. Math. Stat. **53** (2023), 251-268.
- [10] T. Sultana, M. Aslam, M. Raftab, *Bayesian estimation of 3-component mixture of Gumbel type-II distributions under non-informative and informative priors*, Journal of the National Science Foundation of Sri Lanka **45** (2017), 287-306.
- [11] L. von Bortkiewicz, *Variationsbreite und mittlerer Fehler*, Berliner Mathematische Gesellschaft, 1922.