

## A HEURISTIC FOR FINDING CHEATING IN CHESS

KJETIL K. HAUGEN<sup>1</sup>, KNUT P. HEEN<sup>2</sup>, DAVID C. SMERDON<sup>3</sup>, TORE NILSSEN<sup>4</sup>

*Abstract.* This article argues by casual empirics that a low draw percent in chess may work as a simplified cheating indicator. Data from a large number of historical chess games (53331) indicate that this extremely simple heuristic may be used as a first test if suspicion of cheating arises for professional chess players. This heuristic does not prove any cheating, but it may be applied as a quick primal indicator of potential cheating behaviour for a player suspected of cheating.

### INTRODUCTION

Chess computer engines (Stockfish, Alpha Zero, etc.) [10, 23, 26] have become so strong over the last 25 years that even the most highly ranked human players have no chances against these opponents<sup>5</sup>. This development has created an opportunity for weaker chess players, even beginners, to use assistance from a chess engine to compete successfully with much stronger players. Using this type of assistance is strictly forbidden in tournament chess, and is considered unethical and dishonourable even in casual, unincentivized games between amateurs. Still, cheating by using an engine is a more serious threat to the game of chess than doping is to any other sport simply because a chess engine is a more potent form of assistance than doping (the chances of victory from using engine assistance are virtually 100%). It is also relatively easy to implement, because even a chess app on a mobile phone is stronger than the best human in the world.<sup>6</sup> Strong cheat detection schemes are therefore important to preserve the game of chess. The quality of these cheat detection schemes was recently questioned when the then-current World Champion, Magnus Carlsen, quit the Sinquefeld Cup in September 2022 after losing to the American Grandmaster Hans Niemann [15], and later accused Niemann of cheating [16]. Niemann admitted to a past history cheating in on-line games but denied any cheating in over-the-board (OTB) games. Chess.com later published a report [6] suggesting that Niemann's cheating on-line was more common than he admitted to and noted that Niemann "is the fastest rising top

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<sup>1</sup> Faculty of Business Administration and Social Sciences, Molde University College, Specialized University in Logistics, Molde, NORWAY. Email: [kjetil.Haugen@himolde.no](mailto:kjetil.Haugen@himolde.no)

<sup>2</sup> Faculty of Business Administration and Social Sciences, Molde University College, Specialized University in Logistics, Molde, NORWAY. Email: [knut.p.heen@himolde.no](mailto:knut.p.heen@himolde.no)

<sup>3</sup> School of Economics, The University of Queensland, Brisbane, AUSTRALIA. Email: [d.smerdon@uq.edu.au](mailto:d.smerdon@uq.edu.au)

<sup>4</sup> Department of Economics University of Oslo, NORWAY. Email: [tore.nilssen@econ.uio.no](mailto:tore.nilssen@econ.uio.no)

<sup>5</sup> Former World Champion, Garry Kasparov, won a match against Deep Blue in 1996 and lost the rematch in 1997 [25]. Chess engines have of course evolved a lot since 1997. As of today, February 2024, the best Chess engine, Stockfish, will beat the world human number one, Magnus Carlsen, in 9957 out of 10000 games. See [21], [4] and [17].

<sup>6</sup> A 50-year-old beginner cannot win a Marathon race against the World Champion just by the assistance of doping, but he can win a chess game against the World Champion of chess with the help of a chess engine. We will return to the issue of doping later on in this section.

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player in classical OTB chess in modern history", strongly insinuating that his cheating is not limited to online chess. The Carlsen/Niemann turmoil seemingly concluded recently with a \$100 billion lawsuit from Carlsen and Niemann against Carlsen and others [1] and a recent dismissal of this lawsuit from the US legal system [13].

The cheat detection schemes used today are what we may call input-based schemes. These are schemes that attempt to monitor and uncover cheating by looking at the player's actions, for example by checking whether the player brings electronic equipment to the venue or comparing player moves vs. engine moves in a particular position. While Chess.com does not reveal the full details of its anti-cheating algorithm, it has in the past explained that its scheme involves a statistical analysis of the likelihood that a player's moves (i.e., player input) were made without engine assistance. A simple output-based cheat detection heuristic, on the other hand, would be a scheme that attempts to identify cheating by looking at the results of the player's games. The Chess.com report is alluding to this when it refers to Niemann's fast rise in classical OTB chess. The purpose of cheating is obviously to improve the player's results (output) rather than to find the best moves in every position (input). The player's results should therefore carry at least some information about whether the player is cheating or not.

In the present paper, we use the Niemann incident as a motivation for a discussion of how a simple output-based heuristic might serve as a first-step in identifying suspicious cheating-like behavior. We do not mean to put forward strong opinions on whether Niemann has cheated beyond what he himself admitted.

In this paper, we propose that a statistically low rate of draws, and a correspondingly high loss rate for a given chess rating-level<sup>7</sup> may suggest cheating. At the extreme, a player who cheats in every game will be detected very easily, because engines are so strong that the cheater will win every game. We are therefore left with the players who cheat in some games, but not all games. These players will tend to win the games in which they cheat and get normal results in the games they do not cheat. The point is that the cheating will help the player achieve a chess rating that is above his actual ability. It is quite common for a game between two players of similar ability to result in a draw<sup>8</sup>, and in fact the frequency of draws increases with the average strength of the players. However, when a cheater plays against a player of similar rating level, the cheater will be a weaker player than the opponent because the cheater's rating is inflated, and the cheater will therefore tend to lose instead of draw (if he does not cheat). This effect should be stronger the more often the cheater cheats because his rating will be (more) inflated relative to his true ability. For example, if a beginner cheats in 70 percent of his games, the beginner will win more than 70 percent of his games but as he gets matched against stronger and stronger players, he will eventually (in the limit) lose all the games in which he does not cheat and end up with a score of 70 percent wins and 30 percent losses and no draws.

In economics, there is a substantial literature on unethical or illegal behavior. Beginning with the seminal work of Nobel laureate Gary Becker [2] about the economics of crime and punishment, such behavior has typically been explained through a model of rational individual response to a decision problem about

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<sup>7</sup> In chess, each player is assigned a rating, which is a measure of the player's chess strength. The difference in two players' ratings gives a predicted outcome, and each player gains or loses points depending on whether the game result is better or worse than this expectation. The most common rating is the Elo rating, see e.g. [9], [7] and [8], so named after its originator Professor Arpad Elo, and which is widely used in other sports as well, such as basketball, baseball, tennis, and even large language models.

<sup>8</sup> For example, the World Chess Championship match in 2016 between Magnus Carlsen and Sergey Karjakin ended in 10 draws, and one win each, before the match finally was decided in a tie-break; two years later, Magnus Carlsen and Fabiano Caruana played 12 draws before the match again was decided in a tie-break. See [27] and [28].

whether to engage in an illegal action.<sup>9</sup> The subsequent consideration of how affected parties (governments, sporting bodies, insurance companies, etc.) react to cheats moves us closer to the principal-agent literature, beginning with the seminal contribution of Ross [20]. The part of the literature dealing with moral hazard is relevant for cheating in chess. The premise in this literature is that it may be very costly to monitor the actions of the player (the agent) for the tournament organizer (the principal). It is often less costly to look at the output of the agent and use that information to infer what action has been taken by the agent. Both premises seem to be true in top level chess. It is difficult to conclude whether a strong chess player came up with a given move by himself or with the help of an engine. Moreover, the results of the games including wins, draws, losses, and number of moves are readily available for information processing. Our more general point is that this information should look somehow different for a cheater than for an honest player. The moral hazard literature has not produced any general rules for how to connect output to input because that connection depends on the situation at hand. Using an engine in chess is different from CEOs playing golf instead of working<sup>10</sup>; the only common denominator is that both types of cheating affect the output. The only general rule that has been established in the moral hazard literature is Holmström's Informativeness Principle, Holmström [14]. The informativeness principle says that including any information incrementally informative about the agent's action will be beneficial. Any cheat detection scheme based on imperfect information will always be fraught with the problem of type 1 and type 2 errors. Some cheaters will not be caught, and some honest players will be accused of cheating. There is no way around that in a world of imperfect information. The point, however, is that adding incrementally informative information reduces the probability of type 1 and type 2 errors. Hence, tougher punishments may be instituted with stronger confidence that the cheater is guilty, which in turn will reduce the overall frequency of cheating. Another branch of Economic Theory, Sports Economics, is also relevant for cheating in chess. Sports Economic Theory includes a fairly rich literature on the subject and will provide some insights in chess-cheating as will soon become evident. Especially, the branch often named as Economics of Doping is relevant here – refer for instance to Berentsen [3]. Analyzing the problem of chess cheating through this lens reveals an interesting peculiarity for the game when compared to other sports [12]. According to Haugen [12], under the simplest possible set of assumptions, doping can be avoided if:

$$\frac{1}{2}a < rc \quad (1.1)$$

or is unavoidable if:

$$\frac{1}{2}a > rc \quad (1.2)$$

In inequalities (1.1) and (1.2),  $a$  denotes the positive utility of winning some contest,  $r$  is the probability of being exposed as a doper/cheater, and  $c$  is the negative utility experienced by the cheater from such exposure. The peculiarity in this case lies in the assumptions. The simple model leading to (1.1) and (1.2) includes, among many other assumptions, that the agent will win the contest with certainty if doping is used. Typically, in general sports, such an assumption is too strong and must be relaxed in better models, which certainly also is the case in [12]. However, in chess,

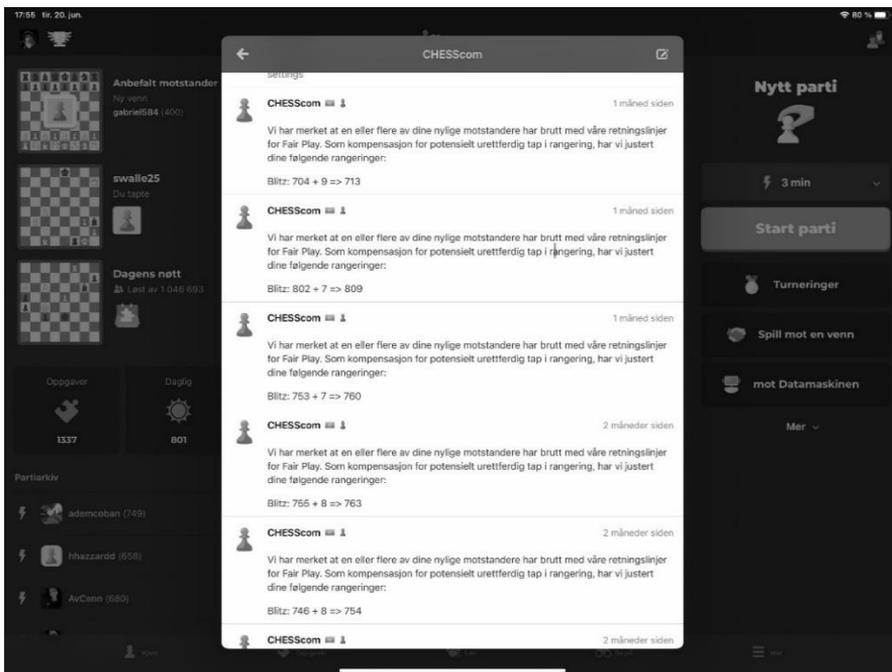
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<sup>9</sup> In recent decades, behavioural economists have proposed an alternative view of cheating and dishonesty to the classical model of Becker, backed by evidence from laboratory experiments. While some of these experimental results have recently come under a cloud [18, 19], others continue to pose interesting questions for the rational model to explain, such as why most people choose to cheat a little bit, but not to the fullest extent.

<sup>10</sup> Playing golf is often used as an example of managerial shirking in economic literature.

with the quality of today’s chess computer software (e.g. the freeware Stockfish<sup>11</sup>), such an assumption of a certain win from cheating is in fact very close to a reality<sup>12</sup>. As such, the simplest possible doping-model seems more applicable in chess than in general sports. Hence, given the size of  $a$ <sup>13</sup> and the small size of both  $c$  and  $r$ , (1.2) should hold. That is, cheating in chess should be quite common.

Another indication supporting such a conclusion is of a more personal nature. One of the authors is actively playing chess on chess.com. This web-based chess platform engages more than 100 million users playing billions of games every day. The choice of engaging here was related to some interest in the game, but also related to writing this paper. Chess.com does in fact inform its users if it suspects (or actually detects and decides) that you have played with a cheater – see figure 1. Although Figure 1 is in Norwegian, it should be possible to understand it. The English translation of the messages simply states that Chess.com has identified that you have played against a cheater and updates your rating (positively) for, in this case, Blitz games. As the Figure also indicates, the frequency of games against cheaters<sup>14</sup> for this simple example indicate at least two cheating opponents per month. A closer inspection actually indicates around one cheating opponent per week. Compared to the number of matches played each week this is perhaps not a big number, but remember that this is detected cheaters, which probably still is a subset of actual cheaters.



**Figure 1.** Chess.com Messages of Identified Cheaters.

<sup>11</sup> Stockfish – see [10] is perhaps the most applied AI chess-engine. Originally developed by Norwegian Tord Romstad, Stockfish holds an ELO rating of 3550, see [26]. If the Stockfish rating is compared to the highest ever human ranking, Magnus Carlsen who reached 2882 in May 2014 [24], Carlsen would be statistically expected to lose 99 games out of every 100 played against Stockfish.

<sup>12</sup> Strictly speaking, this argument is only valid if a single cheater or non-cheaters are playing. A match between two cheaters would most probably end in a draw. However, such outcomes, two cheaters playing, are quite improbable.

<sup>13</sup> According to Green [11], former World Champion Magnus Carlsen earned nearly \$560,000 in 2022.

<sup>14</sup> Chess.com reports comprehensive statistics on cheating, see [5]. For instance, it reports closing more than 550,000 accounts in October 2022 and expects to reach a million closed accounts by early 2024, all caused by cheating. For context, Chess.com currently has over 10 million active member accounts.

Surely, the reader should not be impressed by the ELO ratings here, but that is actually the point. Why should lousy chess players playing Blitz<sup>15</sup> games at chess.com choose to cheat? What could be the rationality behind such actions?

A quick return to the doping models may provide an answer. If we consider the  $c$  in this case, the worst thing that could happen for the cheater is a ban from the site. However, as the membership purely is based on identification through an email address and today's ease to obtain a new one, the only risk you face is to start from scratch again. In this case, this is actually a benefit as your starting rank is 1200, far higher than the levels observed here of 700-800 rating points.

Hence, the cost of cheating at this level is effectively zero. Then, inequality (1.2) is satisfied even with  $a$ 's of epsilon-size.

So, Economics of Doping provide some general insight for the case of chess.

Cheating seems probable, at all quality levels.

The rest of this article is organized as follows: Section 2 formulates and explains our main hypothesis or research question. Section 3 reports on a very simplified empirical analysis, while section 4 concludes and discusses possible enhancements.

## THE BASIC HYPOTHESIS

Before the main research question is formulated, a little more on how chess engines are made is necessary. According to experts we have consulted, it is hard, if not impossible to cheat for a draw. The reason is simple. All chess engines are constructed with the objective of winning chess games. Hence to cheat to achieve a draw is not a rational strategy for a player. Surely, one could use an engine to pick the move which makes the draw probability highest, but this is not necessarily a strategy that maximises the probability of a draw. So, it seems fair to say that a cheating-strategy does not involve the option of a secure draw.

Now, let us set ourselves in the "seat" of a cheater. Under the assumptions of a positive probability of detection and the existence of punishment, a rational cheater should not always cheat. The risk of being detected would prevent such a strategy. Consequently, a situation with cheating in some games and not in others seems reasonable. Furthermore, an identifiable pattern of cheating and non cheating is also not a good idea because this would also lead to a high chance of detection. The actual equilibrium strategy for cheating is consequently not a straightforward problem to solve. Still, we assume (as a first step) that a pattern of randomized cheating and playing fair seems reasonable for a player who decides to cheat.

This leads to an unpredictable set of cheating/no cheating. Or, one set of games where there is no cheating and another (disjoint) set of cheating games. If there is cheating, the outcome would be a certain win, as the above argument secures. If there is no cheating, the fact that the rational player cheats to improve his rating should mean that most of his opponents are better, which will lead to a low percentage of both winning as well as drawn games and a higher share of lost games. Then, it is easy to conclude that the draw percentage will be quite small in both instances. That is, the draw percentage of a cheater ought to be significantly smaller than players who never cheat.<sup>16</sup>

<sup>15</sup> Note that Blitz games are short and should (intentionally) make cheating more difficult.

<sup>16</sup> While the simplified doping theory from section 1 might indicate that everyone should cheat, it is straightforward to extend the model by allowing the cost of cheating,  $c$ , to vary to incorporate individual conscience or moral costs, e.g. by drawing  $c$  from a distribution such that the inequalities hold for an empirically representative share of agents.

This discussion makes it possible to formulate our research question:

**Research Question:** *Cheaters in chess should, over many chess games, have a lower draw share than non-cheaters.*

## THE EMPIRICAL ANALYSIS

The data described in appendix A contains results from 53331 historic chess games taken from the 30 best ranked chess players in March 2023. The data is an aggregate on all formats, ranging from Blitz to classical chess, including on-line as well as OTB chess.

This data is not of such a quality that we can use it to speak to the Carlsen Niemann controversy. Still, it is useful for an illustration of our approach. As can be observed in Table 1, which contains the available data sorted by the lowest draw percentage, Hans Niemann has the lowest. One question that could be of further interest is to check if his draw percent is significantly different from the other players. According to the analysis in appendix B, it is; at the 95% level.

To be sure, this data contains all kind of matches, and in a more ambitious empirical analysis, a lot of variables should have been controlled for – age, rating, tournament types, game speed – slow (‘classical’) games, fast (‘blitz’) games, etc. – to name a few. Still, our purpose with this study was mainly to make a simple game theoretic argument and check if the simplest possible empirical analysis would support it.

## CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

This article suggests a simple heuristical method to indicate who might be cheaters in the game of chess, since cheaters should be expected to draw less often than non-cheaters. We find that Hans Niemann has the lowest draw percentage among top chess players, a fact that might seem to corroborate recent cheating allegations against him. We would like to stress, though, that our method, in the format presented here, is too simplistic to draw conclusions about Hans Niemann.

Clearly, the trending popularity of on-line professional chess tells a story of greater awareness regarding cheating. The popularity of on-line chess platforms like Chess.com may be threatened by increased cheating, just like other sports face similar threats.

A more sophisticated approach than the one presented here is left for future research. Still, we feel that the story told has scientific merit, and may provide new ideas and approaches for both chess players and chess event organizers.

One limitation of our approach is that it may indicate a false positive if the type of player suspected of cheating (but not actually cheating) is correlated with the type of player who scores a low frequency of draws. For example, a player’s playing style might influence both of these traits. While this endogeneity cannot be identified through a cross-sectional analysis, it would be possible to analyse a player’s draw frequency over time, looking for changes in the distribution of a player’s results and how these changes mapped to externally raised cheating suspicions.

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## APPENDIX A. RAW DATA

Table 1 below contains raw data used in this article. All data are collected from open internet sources including chessbase.com and fide.com.

**Table 1.** Games, Wins, Draws, Losses, Draw%, Rating and Age for the the 30 best chess players in the world, March 2023.

<b>Name</b>	<b># Games</b>	<b>Win</b>	<b>Draw</b>	<b>Loss</b>	<b>Draw%</b>	<b>Rating</b>	<b>Age</b>
Niemann	615	268	169	178	27.48%	2706	20
Gukesh	597	268	191	138	31.99%	2730	17
Abdusattorov	695	288	248	159	35.68%	2731	19
Keymer	378	138	135	105	35.71%	2699	19
Firouzja	878	338	319	221	36.33%	2785	20
Jorden van Foreest	1306	501	477	328	36.52%	2681	24
Praggnanandhaa	720	281	267	172	37.08%	2691	18
Duda	1281	502	494	285	38.56%	2724	25
Carlsen	4210	1878	1654	678	39.29%	2852	33
Rapport	1137	446	450	241	39.58%	2745	27
Erigaisi	356	147	141	68	39.61%	2701	27
Caruana	2575	955	1056	564	41.01%	2766	31
Mamedyarov	3085	1126	1298	661	42.07%	2738	48
Sarin	539	200	227	112	42.12%	2676	19
Esipenko	1009	386	426	197	42.22%	2680	21
Nakamura	3227	1289	1388	550	43.01%	2768	36
Sevian	848	329	365	154	43.04%	2687	23
Xiong	971	358	424	189	43.67%	2692	23
Nepomniatchi	2473	891	1092	490	44.16%	2795	33
Dubov	1285	435	586	264	45.60%	2708	27
Vachier Lagrave	3212	1204	1467	541	45.67%	2736	33
Karjakin	2841	987	1316	538	46.32%	2747	33
Aronian	3688	1269	1759	660	47.70%	2745	41
Andreikin	1387	488	682	217	49.17%	2729	33
So	2255	770	1137	348	50.42%	2761	30
Anand	3923	1370	1985	568	50.60%	2754	53
Liren	1471	461	758	252	51.53%	2788	31
Dominguez Perez	1817	517	939	361	51.68%	2738	40
Giri	2435	702	1302	431	53.47%	2768	29
Radjabov	2117	614	1143	360	53.99%	2747	36

## APPENDIX B. STATISTICAL TEST

The obvious test to perform is a Two proportion  $Z$ -test – see for instance [22]. Naturally, a one-sided test is chosen and the formula:

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1 - \hat{p}) \left[ \frac{1}{n_1} + \frac{1}{n_2} \right]}}$$

is used to calculate the  $Z$ -score. Using the data in Table 1 in appendix A, we

find  $\hat{p}_1 = 0.3199$ ,  $\hat{p}_2 = 0.2748$ ,  $n_1 = 597$ ,  $n_2 = 615$  and  $\hat{p} = \frac{169+191}{615+597} = 0.2970$

which gives  $z = 1.71786$ . As the corresponding significance probability (the  $p$ -value) is 0.04291 which is smaller than 0.05, we can claim that Niemann's draw percent is smaller than that of Gukesh (at the 95% level) – the second lowest draw percent. Correspondingly, Niemann's draw percent is also significantly smaller than any other of the 28 players in our data set.

A similar check for Gukesh against Abdusattorov provides data:  $\hat{p}_1 = 0.3568$ ,

$\hat{p}_2 = 0.3199$ ,  $n_1 = 695$ ,  $n_2 = 597$  and  $\hat{p} = \frac{248+191}{695+597} = 0.3398$  and  $z = 1.3961$ . Hence,

Gukesh does not (at the 95% level) have a significantly lower draw percent than Abdusattorov. As most other draw-percent differences are significantly smaller than these two, it feels safe to conclude that Niemann's draw percent is significantly smaller than his neighbours in table 1, as well as all others in the same table. He is also the only player with a significantly lower draw-percent than any other player.