

MULTIFRACTAL MODELLING AND SIMULATION OF SHRAPNEL FRAGMENTATION PATTERNS FROM EXPLOSIVES

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Abstract: This study focuses on multifractal modelling and simulation of shrapnel fragmentation patterns from explosives. Therefore, in this research study, different numerical methods are considered for modelling fragments and blast, which are generated from detonation of explosive. These methods are applied to different scenarios in which explosive devices are improvised within confined spaces. A burn method was used for modelling the detonation of explosives with condensed phase within a 3D multi material flow solver. Initially, an explosion is simulated within a “cylindrical tube and the fluid solver” is validated for a blast which is partially confined. Later on, the explosions were simulated on a subway for C4 charges of 30kg and 10kg. The maximum pulse as well as the overpressure within the shock, are utilized for predicting the risk of the blast injuries. In order to improvise the threats due to explosives, C4 charges were modelled with disk, cylindrical and spherical shapes, surrounded by a spherical fragments layer. It has been observed that charge shape of the explosive plays an essential role in spherical fragments acceleration. Thus, this study has also been beneficial in providing different theoretical as well as practical implications.

1. INTRODUCTION

The detonation of a bomb causes pieces of metal to be propelled outward at high speeds. This process of fragmentation results in the creation of shrapnel, which are metal fragments [24]. The understanding of how these fragments form and travel can provide significant implications for minimizing unintended damage. The fragments have destructive impacts on the surrounding environment, including human and material targets. The impact loading requires comprehensive analysis due to the different types of loading [2]. One of the major threats to military assets

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is explosions. Thus, during the second World War, several analysts analysed the explosions, by studying the distribution of the number and weight of the fragments [11, 16, 23].

Among the analysts, the Gurney equation is a prominent approach which has been utilized by various authors to build computer models [6, 7, 13]. The Gurney energy is a characteristic specific for each explosive. The Gurney energy determines the terminal velocity of the fragments which demonstrates the speed gained by fragments propelled during an explosion. However, despite its popularity, the Gurney equation stands in contradiction to some of the experimental evidence for fragments near the warhead's end plates as noted by Felix, et al. [9]. They also highlighted that the equation for a warhead that is cylindrical is one-dimensional and therefore, it does not take into consideration the fragment velocity for the end plates. Analysts have used the limitation of previous methods to build improved models. For instance, Victor [25] expanded on the work through the adoption of a two-dimensional equation. Furthermore, the current era is characterised by advanced abilities of computer systems with scholars improving estimation accuracy by using tools such as Finite Element Analysis. Nonetheless, extensive resources are required and real-time calculations are not possible [9].

Numerical simulations have been designed and developed to study fragmentation [12]. Nonetheless, there are limitations in the modelling approach in offering accurate predictions of the pattern such as the limitation of the Gurney equation of only one opening velocity [9]. The numerical models can be further enhanced which can improve our understanding of fragment patterns. This information and model simulation are critical to strengthen the design of warheads. Not only the advancement in design, but the study also offers vital information for enhancing safety assessments.

Thus, due to the importance of understanding the dynamics of fragmentation, this study intends to model the fragmentation process. The aim of the study is to improve the precision in the estimation of the initial velocity of fragments. The study leverages on the limitation in previous methods to propose an enhanced approach. Following the methodology proposed by Breech [4], a two-dimensional approach is used to model the explosion. Similar to the models presented by Breech [4] and Gurney [11], the study also applies a physical model for the initial velocity assessment of fragments. The methodology provided in this study takes into consideration the momentum and kinetic energy. Overall, the study has aimed to refine prior models and approaches. Through an extensive approach, we build a model and simulation of shrapnel fragmentation patterns from explosives. In doing so, we present the numerical methodology and techniques in the next sections. After a thorough discussion, conclusions and implications of the study are provided in the end.

2. LITERATURE REVIEW

The "improvised explosive devices IEDs" depicts an increasing global threat. These are usually utilized for sabotage or terrorism. In this regard, even a "small person borne IED (PBIED)" entailing a few kg of explosives can be taken into critical infrastructure or areas which are crowded. It can be exploded with damaging or extremely devastating consequences [1, 5, 10]. To assess and adequately estimate the threats of IED, simulation and modelling can provide meaningful insights. The simple scenarios can be examined with the tools of rapid running engineering on the basis of simplistic models. In this regard, the critical cases may require the

“computational fluid dynamics” (CFD) and imitations of Multiphysics. In this regard, the researchers have studied various numerical simulations for the assessment of detonation concerning small charges of explosives. These can be the representative of PBIED threats. According to Champion, et al. [5], the spontaneous devices are usually crammed with the objects of metal. It involves nails, bolts, nuts, or ball bearing so that the shrapnel can be generated. According to Elsayed [8], these wreckages can enhance the steering power of explosions and may result in wounds at better distance than the over pressure of blast merely. The past assessment or related past studies have revealed that the injuries from secondary blast such as wounds of ballistic resulted due to fragments accounts for the majority of injuries that have been observed among survivors [5, 8]. The injuries including rupture of eardrum or blast lung (pulmonary barotrauma) are also observed commonly in the victims of bombing bearer to explosion. The research by Price, et al. [18] also presented a method for simulating IEDs through which high velocity fragments can be resulted and the injuries resulted due to blast can also be assessed. In this regard, the fundamental numerical arrangements have been authenticated in the past research by Price, et al. [19] and Price, et al. [20]. However, the recent focus is to validate the predictivity competence of such models which have also been studied by many researchers recently.

Huge number of previous studies have focused on modelling explosions in the urban areas. In this regard, Rose and Smith [22] also conducted a research based on a comprehensive simulation and experimental campaign for the propagation of blast in distinct layouts of streets. Their testing also depicting a channelling influence through which the pressure of blast can be increased along with the compulsion when the detonation is limited through a street that is narrow. With the increased advancements in numerical techniques and computerized technologies, it has become likely to run large-scale imitations of the air explosions in geometrics of real city. Lohner, et al. [15] also revealed the results of his study regarding simulations within urban explosions with an impressive element which were based on the cores of disbursed system of computing. The running of these simulations needs contact to big computing schemes. These are not frequently obtainable to the researchers. Research by Rice, et al. [21], depicted that it is possible to attain adequate predictions of the blast from the “courser grid simulations” on notebook computers or even work stations. It can be undertaken by utilizing a “multi-mesh approach” with required attention towards discretization in the critical areas.

The above discussion undertakes large explosions including the vehicle bombs in outdoor environments. However, the threats of PBIED are likely to be smaller charges that are experienced indoor or in the enclosed surroundings. In this regard, the confined spaces result in reflections of multiple internal shocks which results in complicated profiles of pressure wave. This also increases the deadly influence of explosion waves and is linked with advanced extent of injuries [1, 5, 10]. Baker, et al. [3] also discussed the complexities of internalized loadings of blast. It can also be observed in different scenarios including the explosions happening in tunnels [17], underground stations of train and carriages of train etc along with buses [14]. According to, the high impact for terrorists is the train or transit can be established by current occurrences in London 2005 Madrid 2004 and Moscow 2004, 2010 [14]. This highlights a motivation for the scenarios of subway or tunnel explosions that has been studied by Price, et al. [18].

Prediction of hastening of fragments is another central feature of IED model. According to the research by Price, et al. [18], the numerical methods were

developed for the purpose of modelling blast and fragments resulted from the explosive detonation. The explosion of summarised phase explosives was modelled with the technique of program burn in a “three dimensional Multi Material flow solver”. Their research has applied them to such scenarios that represents unplanned volatile devices in the limited spaces. The findings of their research revealed that the “explosive charge shape” plays a significant part in the quickening of round fragments. However, developing better projecting method is valuable for “real life IED scenarios” concerning the upcoming of “emergency response planning”.

3. METHOD

In order to determine the multifractal modelling and simulation of shrapnel fragmentation patterns from explosives, numerical methodology was considered for this study.

3.1 Multimaterial Fluid

The “Euler equations” along with 5-equation model were used for governing the multimaterial flows which are not steadily compressible [26]. These equations are expressed compactly as:

$$\frac{\partial U}{\partial t} + \nabla \cdot F = S \quad \text{e.q. (1)}$$

In this regard, U is stated as the conserved vector, while F is the flux vector and S is the source vector. However, the 5-equation model is presented for 2 materials and can also incorporate other materials by integrating another mass equation as well as transport equation for every incorporated material. Therefore, the total energy and density of the fluid mixture are measured as:

$$\rho = \sum_i z_i p_i \quad \text{e.q. (2)}$$

$$\rho E = \rho e + \frac{1}{2} \rho u^2 \quad \text{e.q. (3)}$$

Here, e is stated as the particular interval energy, while p_i is stated as the phase density within the context of multimaterial mixture. The 5-equation model is also presented within the context of “Mie-Gruneisen equation of state (EOS)” as:

$$p(\rho, e) = \Gamma(\rho) [\rho e - \rho e_{\text{ref}}(\rho)] + p_{\text{ref}}(\rho) \quad \text{e.q. (4)}$$

The EOS (ideal gas) was used for explosive detonation simulations within the context of air, while shock-wave EOS was used for explosives which were unreactive and “Jones-Wilkins-Lee (JWL) EOS” was used for detonation products. JWL EOS as well as ideal gas were considered to be effective for modelling materials under higher pressure.

3.2 Particles

According to Newton’s law, the particles’ motion is determined as follows:

$$\frac{dx_p}{dt} = u_p \quad \text{e.q. (5)}$$

$$m_p \frac{du_p}{dt} = F_{qs} + F_{pg} + F_{am} \quad \text{e.q. (6)}$$

Here, m_p is stated as the mass, while x_p is the position and u_p is stated as the particle velocity. However, the particle forces incorporate F_{pg} (pressure-gradient), F_{am} (added-mass forces) and F_{qs} (quasi-steady drag). Additionally, the work transfer and particle momentum are also evaluated as stated by Price, et al. [18].

3.3 Temporal and Spatial Discretization

The Euler equations within the context of fluid flow are solved, using edge-based and a vertex-centred finite volume process on unstructured grids. A robust “Delaunay mesh generation method” was used for decentralizing 3D computational domain. Primitive variables are used for computing gradients via a “Green-Gauss method” and the pressure oscillations are avoided at the interface. “Courant-Friedrichs-Lewy” situation was used for calculating time step.

3.4 “1D Multimaterial Solver”

This solver represents the cylindrical, spherical, and planar symmetry situations via the utilization of geometric sources. The solutions obtained from this solver can also be utilized for initializing 3D explosions and other air-blast models.

3.5 Evaluation of Blast Risk

“Blast severity” is often stated as overpressure’s peak values and maximum impulse of significant phase. In context of risk analysis, impulse and insignificant phase pressure are often neglected, as they are less likely to cause any injury. Therefore, within the context of confined spaces, explosions can result in secondary shocks which can promote “significant phase impulse.” Thus, maximum impulse and peak overpressure are determined as follows:

$$p_m = \max(p(t) - p_0) \quad \text{e.q. (7)}$$

$$\forall t \in [0, t_{sim}]$$

$$I_m = \max \left(\int_0^t [p(t) - p_0] dt \right) \quad \text{e.q. (8)}$$

$$\forall t \in [0, t_{sim}]$$

For this study, “the adjusted severity of injury index (ASII)” was also determined. For this purpose, the lethality of greater than or equal to 50% was also considered, in association with $ASII \geq 3.6$.

4. RESULTS

Table 4.1 shows the JWL EOS for explosive materials for three types of explosives, used for this study. The value of ω for TNT, C4 and m/46 was found to be 0.31, 0.26 and 0.28.

Table 4.1. JWL EOS for explosive materials.

Explosive	ρ_0 (kg/ m ³)	ω	A (gigapascal)	B (gigapascal)	R_1	R_2	DCJ (m/ s)	p_{CJ} (gigapascal)	E_{det} (megajoule/ kg)
TNT	1631	0.31	372.3	3.232	4.14	0.94	6931	21.1	4.293
C4	1602	0.26	609.7	12.94	4.51	1.41	8192	28.1	5.622
m/46	1501	0.28	759.8	12.55	5.11	1.51	7681	21.24	4.701

In the current study, the test was based on the detonation experiments within a cylindrical steel tube as shown in figure 4.1. the diameter of the tube was 1.5m and its length was 24 m. One end of the tube was closed, while a 0.5 kg spherical

explosive was detonated within the tube's centre at 1 m from the end that was open. The external volume was also integrated within the model at the cylinder's open end. Pressure was also measured at 12 m as well as 21 m of the cylinder wall, from the tube's open end.

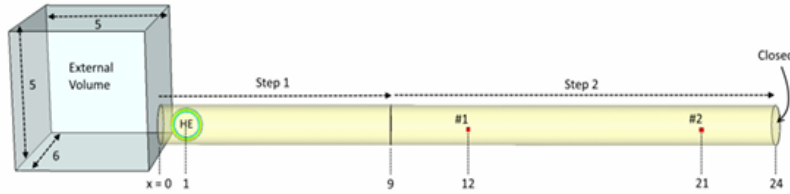


Figure 4.1. Model for tube explosion.

Modelling of charge of m/ 46 is done with “burn model” as well as JWL EOS parameters, which are presented in table 4.1. The 1D multimaterial solver is utilized for providing initial detonation solution, which is interpolated within step 1 domain. In this regard, 1D simulation was found to be resolved, incorporating a 1 mm cell size and 0.75m shock. In 1 and 2 steps, a 0.030 m cell size is also prescribed up to 0.750 m. Step 1 solutions are integrated within step 2 full domain after 10 ms. Figure 4.2 represents the pressure contours within the cut plane at step 1 end.

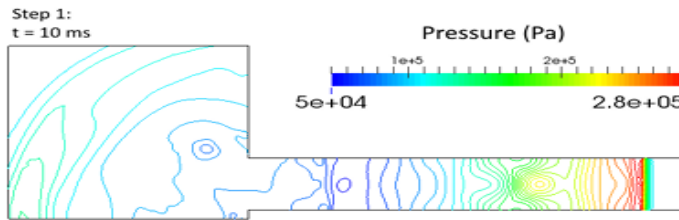


Figure 4.2. Tube explosive (t= 10 ms, step 1).

From the simulations, overpressures were compared with the attained test data for both gauge locations as shown in figure 4.3. A low-pass filter was used for processing the experimental data to decrease the noise. Different shock reflections were observed from the associated tube wells after detonation. In figure 4.4, streak diagrams are also observed within the context of simulations, utilizing a line which runs throughout the axis of tube via the domain, within the context of shock dynamics.

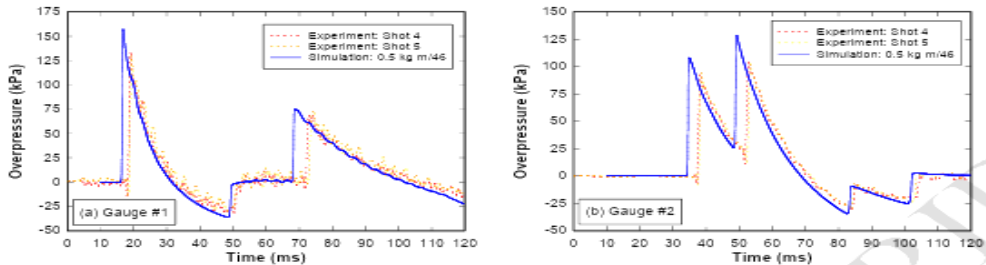


Figure 4.3. Experimental and Simulation Pressure (a) 12.0 m, (b) 21.0 m from the tube's open end.

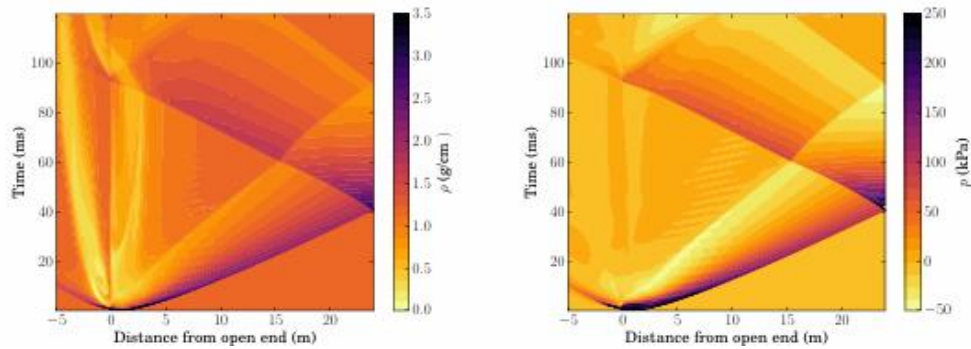


Figure 4.4. “Streak diagrams” (a) shows density and (b) shows pressure at tube axis.

Numerical method was also utilized for simulation method application for assessing the risks of explosions on infrastructures of land mass transport. Explosive C4 charges of 30 kg and 10 kg were selected to present “briefcase bombs.” The simulation details for the associated explosive were also considered for this, presented in table 4.2. Simulation was found to have two steps. A decreased domain size was observed in step 1, allowing a “finer mesh” for calculating the associated detonation. Near the explosive, the cell size was recorded to be 8.0 mm, while the size of the background increases to 80 mm. However, step 2 of the simulation has a 30 mm cell size, the background size was increased to 100 mm. Moreover, at step 2 end, the reflected pressures are decreased to ambient situations.

Table 4.2. Simulation details (C4- 10kg explosion at subway).

Step no.	n _{el}	n _{CPU}	t (ms)	t _{sol} (hr)
1	4.351×10 ⁶	10.0	0 – 0.61	2.21
2	4.231×10 ⁶	10.0	0.6 – 4.1	~24

Figure 4.5 shows that eardrum rupture probability within the context of 10 kg of explosive in subway. Four probability regions are indicated in this regard. The risk region of greater than 90% eardrum rupture risk charge covers about 6.51 m from the associated explosive. Further, the risk of the explosive decreases by 50%.

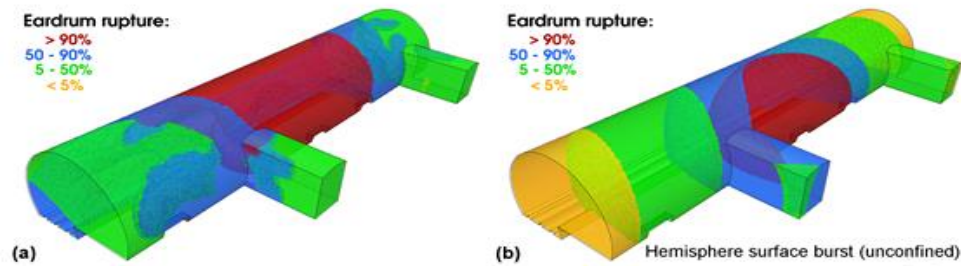


Figure 4.5. Eardrum rupture probability at subway explosion.

The aim of this study was to determine the “Improvised explosive devices” (IED) fragmentation acceleration for various charge geometries. The 3D simulations incorporate “symmetric configurations of disk, cylindrical and spherical-shaped C4 explosive charges,” which are also surrounded by spherical fragments layer.

Figure 4.6 shows various configurations. 3 symmetry planes were used for decreasing computational costs.

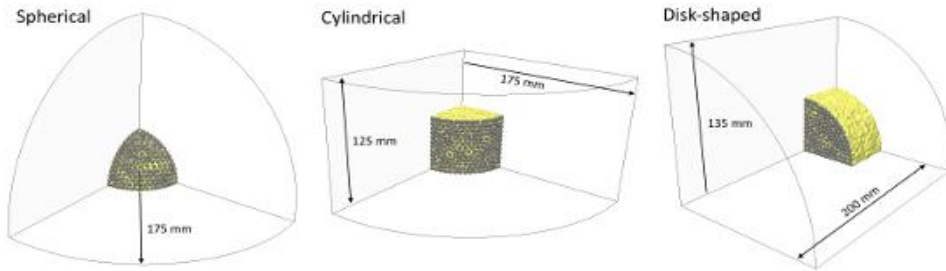


Figure 4.6. 3 utilized symmetry planes.

Table 4.3 shows the $D/2$ (Explosive radius), m_c (explosive mass), $L/2$ (half length of explosive), n_p (Lagrangian particles no.) and total mass of particle (P_{mp}) of the associated explosives.

Table 4.3. Shape simulations in “IED charge”

Shape of charge	$D/2$ (in mm)	$L/2$ (in mm)	m_c (in gm)	n_p	(in gm)
Sph	50.0	-	105.0	167.0	85.0
Cyl-shape (Length/Diameter = 1.1)	50.0	50.0	157.0	173.0	88.0
Cyl-shape (Length/Diameter = 1.21)	50.0	60.0	188.0	205.0	105.0
Dsk-shape (Length/Diameter = 1.1)	50.0	50.0	157.0	83.0	42.0
Dsk-shape (Length/Diameter = 0.832)	60.0	50.0	226.0	125.0	64.0

Table 4.4 shows the configurations ratio of C/M , velocities of the particles and Gurney velocities. Two of the Gurney velocities are presented, which assume disks' infinite diameter ($V_{G, \infty}$), length of cylinders and end losses of equations. It has been observed that C/M has increased between cylinder, disk, and sphere charges.

Table 4.4. Charge shape of IED and Gurney equation results.

Shape of charge	C/M	$V_T(\psi = 1)$	$V_{G, \infty}$	$V_{G, losses}$	$\frac{V_T(\psi = 1)}{V_{G, \infty}}$
Sph	1.22	1.22	2.24	2.24	0.53
Cyl-shape (Length/Diameter = 1.0)	1.77	1.34	2.61	2.11	0.52
Cyl-shape (Length/Diameter = 1.2)	1.81	1.36	2.62	2.18	0.53
Disk-shape (Length/Diameter = 1.0)	3.72	1.42	3.44	1.98	0.42
Disk-shape (Length/Diameter = 0.832)	3.54	1.54	3.43	2.08	0.44

5. DISCUSSION AND CONCLUSION

The recent research was undertaken to evaluate the multifractal modelling and simulation of shrapnel fragmentation patterns from explosives. The present research has revealed the competences of a “coupled Euler Lagrange” technique of simulation for the identification and assessment of risks resulted due to the threats of IED in representative states. The enhancing potential of personal computers have made it feasible to operate more detailed and larger simulations which are based on physics. These can play an imperative role in contradicting the increasing hazard of the violence and IEDs. The problem solver of CFD has been authenticated through utilization of the data from the detonation within a

“partially confined cylinders”. Within the context of “tunnel like geometry”, the detonation persistently changed in a one-dimensional type of shock wave that is more influential as compared to the unconfined blast in air. Two cases of detonations on the “subway train platform” were replicated. In this regard, forecast peak compression and impulse were undertaken for the development of 3D visualization concerning the probabilities of predicted injuries in the domain. A complex blast profile of wave was produced through the enclosed geometry with the pulse gradually growing over a period of “tens of milliseconds”. Along with the minor 10kg, charge of C4, the level of injuries related to fatal lung forecasted by probit calculations “based on the Bowen curves” were much smaller as compared to the method of Axelsson. This underscores and emphasizes the requirement for more authentication and better injury of human models that can be implemented in the simulation of blasts. Through the coupling of CFD solvers, IED fragments were modelled with a Lagrangian approach of point-particle. The geometry of IED significantly affected the terminal velocities and acceleration of fragments adjacent to the charge. The particles surrounding disk and cylindrical charges “both with $L/D=1,0$ ” possessed terminal velocities that are 16 percent and 10 percent greater correspondingly than a round charge with the similar thickness of operative charge “albeit distinct C/M ratios”. The velocities of fragment forecasted through equations of Gurney equations end losses were greater as compared to the simulations by 40 percent to 55 percent. This suggests that a huge proportion of explosion energy is not transmitted to the metal because of the gaps among particles. The numerical imitation of IED explosion on the disposal of a bomb specialist reveals the capability to evaluate risks from fragment and shock loadings in the nearer fields. Although the suits of EOD are formulated to endure near detonation range, the IEDs produces greater fragments of speed which might increase the rating of suits ballistics. In this regard, development of adequate and better analytical tools for the situations of real-life IED is appreciated for the future of ERP “emergency response planning”. The future researchers can also avail the opportunity to investigate enhancing strength of the particle technique, for huge particles and small charges. It can also aid in extension of detonation risk assessment models for the scenarios of packed surroundings. This study has focused on developing numerical techniques for the fragments and modelling blast resulted from the volatile explosion and implement them to the scenarios showing improvised volatile devices in the spaces which are confined. The shortened phase explosive’s detonation has been modelled with the method of program burn in a “three-dimensional multi-material flow solver”. However, the solver has been combined with a Lagrangian particle solver so that the acceleration of explosive-driven fragments can be modelled. In this regard, an explosion in a lengthy cylindrical tube was first simulated for the validation of the fluid solver in a “partially confined blast”. Afterwards a simulation explosion was undertaken on the subway train platform for the C4 charges of 10 kilograms and 30 kilograms. The extreme shock overpressure and impulse were utilized for the prediction of risk of the common injuries resulted due to blast. For the representation of improvised explosion threats C4 charges was modelled with the cylindrical, spherical and disk shapes that was protected and covered through a layer of spherical fragments. This study has concluded that the volatile charge shape plays a crucial part in the quickening of round fragments. Moreover, a practical situation of an “improvised explosive detonation” near the technician of a bomb is examined to evaluate the route of fragment and loads of blast in the closer field.

6. RESEARCH IMPLICATIONS

This study includes both practical as well as theoretical implications. For instance, this study has been effective in presenting the significance of Gurney equation in building computer models for multifractal modelling and simulation of shrapnel fragmentation patterns from explosives. This study has also been effective in improving the literature concerning the simulation of shrapnel fragmentation patterns, within the context of explosives. Moreover, this study has also been efficient in determining the role of blasts in causing various common injuries, encouraging the associated stakeholders to design and develop explosives, focusing on their impact on the eardrum fracture and other related injuries. This can lead to the development of IEDs, leading to significant outcomes.

7. LIMITATIONS AND FUTURE RESEARCH

Every research study has its own pros and cons. Similarly, the current research also integrates a few shortcomings which are highlighted in this section. First, ineffective predictive tools were used for developing real-life IED scenarios for this study, due to limited resources. Second, not much focus has been given on practical implications due to shorter timeframe and final, no effective evaluation models were incorporated for determining the blast risk.

Therefore, future research can focus on overcoming these shortcomings. For instance, effective predictive tools can be used in the future for determining real-life IED situations. Additionally, effective response planning can also be done for future explosive emergency. In this regard special focus can also be given on the particle method, focusing on smaller charges and larger particles. Moreover, effective risk evaluation models can also be incorporated in future research, especially for crowded areas to identify effective solutions.

REFERENCES

- [1] J. L. Arnold, P. Halpern, M.-C. Tsai, H. Smithline, *Mass casualty terrorist bombings: a comparison of outcomes by bombing type*, *Annals of Emergency Medicine* **43** (2004), 263-273.
- [2] O. Atoui, G. Kechagiadakis, A. Moumen, A. Maazoun, B. Belkassem, L. Pyl, D. Lecompte, *An explosive driven shock tube-based laboratory scale test for combined blast and fragment impact loading*, *Applied Sciences* **12** (2022), 6854.
- [3] W. E. Baker, P. A. Cox, J. J. Kulesz, R. A. Strehlow, P. S. Westine, *Explosion hazards and evaluation*, Elsevier, 2012.
- [4] B. A. Breech, *Extension of the gurney equations to two dimensions for a cylindrical charge*, Army Research Laboratory, 2011.
- [5] H. R. Champion, J. B. Holcomb, L. A. Young, *Injuries from explosions: physics, biophysics, pathology, and required research focus*, *Journal of Trauma and Acute Care Surgery* **66** (2009), 1468-1477.
- [6] C. Choi, M. Callaghan, P. van der Schaaf, H. Li, B. Dixon, *Modification of the Gurney equation for explosive bonding by slanted elevation angle*, Australia: Land Division DSTO Defence Science and Technology Organisation, 2014.
- [7] L. A. da Silva, S. Johnson, R. Critchley, J. Clements, K. Norris, C. Stennett, *Experimental fragmentation of pipe bombs with varying case thickness*, *Forensic Science International* **306** (2020), 110034.

- [8] N. M. Elsayed, *Explosion and blast-related injuries: effects of explosion and blast from military operations and acts of terrorism*, Academic Press, 2010.
- [9] D. Felix, I. Colwill, E. Stipidis, *Real-time calculation of fragment velocity for cylindrical warheads*, Defence Technology **15** (2019), 264-271.
- [10] R. Golan, D. Soffer, A. Givon, K. Peleg, I. T. Group, *The ins and outs of terrorist bus explosions: injury profiles of on-board explosions versus explosions occurring adjacent to a bus*, Injury **45** (2014), 39-43.
- [11] R. W. Gurney, *The initial velocities of fragments from bombs, shell, grenades*, Ballistic Research Laboratories Aberdeen Proving Ground, MD, 1943.
- [12] K. T. Hughes, S. Balachandar, A. Diggs, R. Haftka, N. H. Kim, D. Littrell, *Simulation-driven design of experiments examining the large-scale, explosive dispersal of particles*, Shock Waves **30** (2020), 325-347.
- [13] X. Kong, W. Wu, J. Li, F. Liu, P. Chen, Y. Li, *A numerical investigation on explosive fragmentation of metal casing using Smoothed Particle Hydrodynamic method*, Materials & Design **51** (2013), 729-741.
- [14] M. Larcher, F. Casadei, G. Solomos, *Risk analysis of explosions in trains by fluid-structure calculations*, Journal of Transportation Security **3** (2010), 57-71.
- [15] R. Lohner, F. Camelli, J. Baum, *Large-scale blast calculations on GPU clusters*, in: 50th AIAA Aerospace Sciences Meeting including the New Horizons Forum and Aerospace Exposition, American Institute of Aeronautics and Astronautics, 2012, 565.
- [16] N. F. Mott, *A Theory of the Fragmentation of Shells and Bombs*, Ministry of Supply, in: Shock Wave and High Pressure Phenomena, Springer, Berlin, Heidelberg, 1943, 243-294.
- [17] O. Pennetier, M. William-Louis, A. Langlet, *Numerical and reduced-scale experimental investigation of blast wave shape in underground transportation infrastructure*, Process Safety and Environmental Protection **94** (2015), 96-104.
- [18] M. A. Price, V.-T. Nguyen, O. Hassan, K. Morgan, *An approach to modeling blast and fragment risks from improvised explosive devices*, Applied Mathematical Modelling **50** (2017), 715-731.
- [19] M. A. Price, V.-T. Nguyen, O. Hassan, K. Morgan, *A method for compressible multimaterial flows with condensed phase explosive detonation and airblast on unstructured grids*, Computers & Fluids **111** (2015), 76-90.
- [20] M. A. Price, V. T. Nguyen, O. Hassan, K. Morgan, *An Euler-Lagrange particle approach for modeling fragments accelerated by explosive detonation*, International Journal for Numerical Methods in Engineering **106** (2016), 904-926.
- [21] D. L. Rice, J. D. Baum, F. Togashi, R. Löhner, A. Amini, *First-Principles Blast Diffraction Simulations on a Notebook: Accuracy, Resolution and Turn-Around Issues*, in: Proc. MABS-20 Conf., Oslo, Norway, September, 2008.
- [22] T. A. Rose, P. D. Smith, *Influence of the principal geometrical parameters of straight city streets on positive and negative phase blast wave impulses*, International Journal of Impact Engineering **27** (2002), 359-376.
- [23] G. I. Taylor, *Analysis of the explosion of a long cylindrical bomb detonated at one end*, Mechanics of Fluids, Scientific Papers of G.I. Taylor **2** (1941), 277-286.
- [24] J. T. Thurman, *Practical bomb scene investigation*, CRC Press, 2017.
- [25] A. C. Victor, *Warhead performance calculations for threat hazard assessment*, in: Department of Defense Explosives Safety Seminar, Las Vegas, Defense Technical Information Center, 1996, 2-26.
- [26] N. P. Weatherill, O. Hassan, *Efficient three-dimensional Delaunay triangulation with automatic point creation and imposed boundary constraints*, International Journal for Numerical Methods in Engineering **37** (1994), 2005-2039.