

SOME PROPERTIES OF FUZZY COMPACT SPACES ON FUZZY IDEAL

HASSAN ABID AL HADI AHMED¹

Abstract: The purpose of this paper is to investigate fuzzy $\tilde{I}$ -compact spaces and study their properties fuzzy compactness modulo a fuzzy ideal. Some of the outcomes in fuzzy compact spaces have been undetermined by consequences in the fuzzy $\tilde{I}$ -compact spaces.

1. INTRODUCTION AND PRELIMINARIES

The idea of fuzzy set was proposed by Goguen [3]. Chang introduced the fuzzy topological space [1]. A fuzzy ideal $\tilde{I}$ on a fuzzy topological space $(\tilde{X}, \tilde{\tau})$ is a fuzzy nonempty collection of fuzzy subsets of $\tilde{X}$ which satisfies:

- (i) $\mu\tilde{A} \leq \mu\tilde{I}$ and $\mu\tilde{B} \leq \mu\tilde{A}$ implies $\mu\tilde{B} \leq \mu\tilde{I}$ (heredity)
- (ii) $\mu\tilde{A} \leq \mu\tilde{I}$ and $\mu\tilde{B} \leq \mu\tilde{I}$ implies $\max\{\mu\tilde{A}, \mu\tilde{B}\} \leq \mu\tilde{I}$ (fuzzy finite additivity).

“We denote a fuzzy topological space $(\tilde{X}, \tilde{\tau})$ with an fuzzy ideal $\tilde{I}$ defined on $\tilde{X}$ by $(\tilde{X}, \tilde{\tau}, \tilde{I})$. An fuzzy ideal $\tilde{I}$ is said to be fuzzy condense or a fuzzy boundary ideal if $\min\{\mu\tilde{\tau}, \mu\tilde{I}\} = \{\tilde{\varphi}\}$.” If $\mu\tilde{A} < \mu\tilde{X}$, $\text{cl}(\tilde{A})$ will denote the fuzzy closure of $\tilde{A}$ in $(\tilde{X}, \tilde{\tau})$. A fuzzy subset $\tilde{A}$ of a fuzzy space $(\tilde{X}, \tilde{\tau})$ is said to be fuzzy $\tilde{g}$ -closed if $\mu\text{cl}(\tilde{A}) \leq \mu\tilde{U}$, whenever $\mu\tilde{A} \leq \mu\tilde{U}$ and $\mu\tilde{U} \leq \mu\tilde{\tau}$. Every fuzzy closed set is fuzzy $\tilde{g}$ -closed but converse is not true. If $(\tilde{X}, \tilde{\tau}, \tilde{I})$ is an fuzzy ideal space, $(\tilde{Y}, \tilde{\sigma})$ is a fuzzy topological space and $\tilde{f}: (\tilde{X}, \tilde{\tau}, \tilde{I}) \rightarrow (\tilde{Y}, \tilde{\sigma})$ is a function, then $\tilde{f}(\tilde{I}) = \{\tilde{f}(\tilde{I}_1): \tilde{I}_1 \leq \tilde{I}\}$ is an fuzzy ideal of $\tilde{Y}$. If $\tilde{I}$ is fuzzy ideal of fuzzy subsets of $\tilde{X}$ and $\tilde{Y}$ is fuzzy subset of $\tilde{X}$, then $\tilde{I}_Y = \{\min\{\mu\tilde{Y}, \mu\tilde{I}_1\}: \mu\tilde{I}_1 \leq \mu\tilde{I}\}$ is an fuzzy ideal of fuzzy subsets of $\tilde{Y}$ [6]. In fuzzy ideal space $(\tilde{X}, \tilde{\tau}, \tilde{I})$, the collection $\tilde{\beta}(\tilde{I}, \tilde{\tau}) = \{\mu\tilde{U} - \mu\tilde{I}_1: \mu\tilde{U} \leq \mu\tilde{\tau}, \mu\tilde{I}_1 \leq \mu\tilde{I}\}$ is a fuzzy basis for a fuzzy topology $\tilde{\tau}_*(\tilde{I})$ finer than $\tilde{\tau}$. When no ambiguity is present, we denote $\tilde{\beta}(\tilde{I}, \tilde{\tau})$ by $\tilde{\beta}$ and $\tilde{\tau}_*(\tilde{I})$ by $\tilde{\tau}_*$. Let $(\tilde{X}, \tilde{\tau}, \tilde{I})$ be an fuzzy ideal space. Then $\tilde{A}_*(\tilde{I}, \tilde{\tau}) = \{\mu\tilde{x} \leq \mu\tilde{X}: \min\{\mu\tilde{A}, \mu\tilde{U}\} \geq \mu\tilde{I} \text{ for every } \mu\tilde{U} \leq \mu\tilde{\tau}(\tilde{x})\}$ where $\mu\tilde{\tau}(\tilde{x}) = \{\mu\tilde{U} \leq \mu\tilde{\tau}: \mu\tilde{x} \leq \mu\tilde{U}\}$. When there is no ambiguity, we will write $\tilde{A}_*$ for $\tilde{A}_*(\tilde{I}, \tilde{\tau})$ and call it the “fuzzy local function of $\tilde{A}$ ”. “The fuzzy simplest

¹ Department of Mathematics, College of Science University of Diyala, Iraq.
 Email: Hassanabdelhadi@uodiyala.edu.iq

ideals are $\{\tilde{\varphi}\}$ and $\tilde{\varphi}(\tilde{X}) = \{\tilde{A}: \mu\tilde{A} \leq \mu\tilde{X}\}$. Observe that $\tilde{A}^* (\{\tilde{\varphi}\}) = \text{cl}(\tilde{A})$ and $\tilde{A}^* (\tilde{\varphi}(\tilde{X})) = \tilde{\varphi}$ for every $\mu\tilde{A} \leq \mu\tilde{X}$."

Note. $\mu\tilde{X} \geq \mu\tilde{A}^*$ if and only if $\min \{(\mu\tilde{U} - \mu\tilde{J}), \tilde{A}\} = \tilde{\varphi}$, when $\mu\tilde{U} \leq \mu\tilde{\tau}(\tilde{x})$ and $\mu\tilde{J} \leq \tilde{I}$.

Lemma 1.1. Let $(\tilde{X}, \tilde{\tau}, \tilde{I})$ be an fuzzy ideal space and let $\tilde{A}$ be a fuzzy subset of $\tilde{X}$. Then:

- (i) $\mu\tilde{A}^* = \mu\tilde{\text{cl}}(\tilde{A}^*) \leq \mu\tilde{\text{cl}}(\tilde{A})$;
- (ii) $\mu\tilde{A}$ is fuzzy $\tilde{\tau}^*$ -closed if and only if $\mu\tilde{A}^* \leq \mu\tilde{A}$ [4].

2. BASIC CONCEPTS OF FUZZY SET

Definition (2-1) [3]:- let $\tilde{x}$ be a non – empty set a fuzzy set $\tilde{A}$ in $\tilde{X}$ is characteriaer by a membership function $\mu\tilde{A}: \tilde{x} \rightarrow [0,1]$ and we can write this fuzzy set as.

$$\tilde{A} = \{(x, \mu\tilde{A}(x): x \in X, \mu\tilde{A}(x) \leq 1\}$$

The collection of all fuzzy sets in $\tilde{X}$ will be denoted by $I^{\tilde{x}}$, when $I^{\tilde{x}} = \{\tilde{A}: \tilde{A} \text{ is fuzzy set in } x\}$

Definition (2-2) [3]:- The support of a fuzzy set $\tilde{A}$ is the set of all $x \in X$ such that $\mu\tilde{A}(x) > 0$ and is denoted by $S(\tilde{A})$.

Definition (2-3): - A fuzzy point $\tilde{P}_x^r$ in x is a special fuzzy set with membership function defined by $\tilde{P}_x^r(y) = \begin{cases} r, & \text{if } x = y \\ 0, & \text{if } x \neq y \end{cases}$

When $0 < r \leq 1$, y is the support of $\tilde{P}_x^r(x)$.

Definition (2-4) [3]:- A fuzzy set $\tilde{A}$ is said to be finite fuzzy set if $S(\tilde{A})$ is a finite set .

Remark (2-5) 1) A non –empty set x is a fuzzy set with membership $\mu_x(x) = 1, \forall x \in X$ and x is called a crisp set.

2) A membership function $\mu_{\tilde{\emptyset}}(x) = 0, \forall x \in X$ is called an empty set and denoted by $\tilde{\emptyset}$.

Definition (2-6) [3]:- Let $\tilde{P}_x^r$ be a fuzzy point and $\tilde{C}$ be a fuzzy set in non- empty set $\tilde{X}$. Then $\tilde{P}_x^r$ is said to be in $\tilde{C}$ or $\tilde{C}$ contains $\tilde{P}_x^r$ if $\mu\tilde{P}_x^r \leq \mu\tilde{C}(x)$ for all $x \in X$ and denoted by $x \in S(\tilde{C})$ [7].

Definition (2-7) [3]:- Let $\tilde{A}$ and $\tilde{B}$ by fuzzy sets of a universal set $\tilde{X}$ then

- 1) $\tilde{A} \subseteq \tilde{B}$ if and only if $\mu_{\tilde{A}}(x) \leq \mu_{\tilde{B}}(x), \forall x \in X$
- 2) $\tilde{A} = \tilde{B}$ if and only if $\mu_{\tilde{A}}(x) = \mu_{\tilde{B}}(x)$ for all $x \in X$
- 3) $\tilde{A}^c$ is the complement of a fuzzy set $\tilde{A}$ with membership function $\mu_{\tilde{A}^c} = 1 - \mu_{\tilde{A}}(x)$
- 4) $\tilde{C} = \tilde{A} \cup \tilde{B}$ if and only if $\mu_{\tilde{C}}(x) = \max \{ \mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x) \}, \forall x \in X$
- 5) $\tilde{D} = \tilde{A} \cap \tilde{B}$ if and only if $\mu_{\tilde{D}}(x) = \min \{ \mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x) \}, \forall x \in X$
- 6) More generally, for a family of fuzzy sets $\{\tilde{A}_\alpha: \alpha \in \Lambda \text{ where } \Lambda \text{ is the any index set}\}$ the union $\tilde{C} = \bigcup_{\alpha \in \Lambda} \tilde{A}_\alpha$ and the intersection $\tilde{D} = \bigcap_{\alpha \in \Lambda} \tilde{A}_\alpha$ and defined respectively by $\mu_{\tilde{C}}(x) = \sup_{\alpha \in \Lambda} \{ \mu_{\tilde{A}_\alpha}(x); x \in X \}$
 $\mu_{\tilde{D}}(x) = \inf_{\alpha \in \Lambda} \{ \mu_{\tilde{A}_\alpha}(x); x \in X \}$

Note. $\mu\tilde{X} \geq \mu\tilde{A}^*$ if and only if $\min \{(\mu\tilde{U} - \mu\tilde{J}), \tilde{A}\} = \tilde{\varphi}$, when $\mu\tilde{U} \leq \mu\tilde{\tau}(\tilde{x})$ and $\mu\tilde{J} \leq \tilde{I}$.

Lemma 1.1. Let $(\tilde{X}, \tilde{\tau}, \tilde{I})$ be an fuzzy ideal space and let $\tilde{A}$ be a fuzzy subset of $\tilde{X}$. Then:

- (iii) $\mu\tilde{A}^* = \mu\tilde{\text{cl}}(\tilde{A}^*) \leq \mu\tilde{\text{cl}}(\tilde{A})$;
- (iv) $\mu\tilde{A}$ is fuzzy $\tilde{\tau}^*$ -closed if and only if $\mu\tilde{A}^* \leq \mu\tilde{A}$.

Proof. Let $\tilde{f}: \mu\tilde{X} \rightarrow \mu\tilde{Y}$ be any continuous map, where $(\tilde{X}, \tilde{\tau}, \tilde{I})$ is fuzzy $\tilde{I}$ -compact. Let $\{\tilde{U}_\lambda: \mu\tilde{\lambda} \leq \mu\tilde{\Lambda}\}$ be an fuzzy open covering of the fuzzy set $\tilde{f}(X)$ by fuzzy sets open in $\tilde{Y}$. Since $\tilde{f}$ is continuous, the collection $\{\mu\tilde{f}^{-1}(\tilde{U}_\lambda): \mu\tilde{\lambda} \leq \mu\tilde{\Lambda}\}$ is an fuzzy open covering of $\tilde{X}$. Given that $\tilde{X}$ is fuzzy $\tilde{I}$ -compact, there exists a fuzzy finite subset $\tilde{\Lambda}^*$ of $\tilde{\Lambda}$ such that $\mu\tilde{X} - \max \{[\mu\tilde{f}^{-1}(\tilde{U}_\lambda): \mu\tilde{\lambda} \leq \mu\tilde{\Lambda}^*]\} \leq \tilde{I}$. Now, $\tilde{f}(\mu\tilde{X} - \max \{[\mu\tilde{f}^{-1}(\tilde{U}_\lambda): \mu\tilde{\lambda} \leq \mu\tilde{\Lambda}^*]\}) \leq \tilde{f}(\tilde{I})$. We know $\mu\tilde{f}(X) - \mu\tilde{f}(\max \{[\mu\tilde{f}^{-1}(\tilde{U}_\lambda): \mu\tilde{\lambda} \leq \mu\tilde{\Lambda}^*]\}) < \tilde{f}(\mu\tilde{X} - \max \{[\mu\tilde{f}^{-1}(\tilde{U}_\lambda): \mu\tilde{\lambda} \leq \mu\tilde{\Lambda}^*]\})$.

$(\tilde{U}_\lambda): \mu\tilde{\lambda} \leq \mu\tilde{\lambda}^* \}$). This implies $\mu\tilde{f}(X) - \mu\tilde{f}(\max \{[\mu\tilde{f}^{-1}(\tilde{U}_\lambda): \mu\tilde{\lambda} \leq \mu\tilde{\lambda}^*]\}) \leq \tilde{f}(\tilde{I})$, since $\tilde{f}(\tilde{I})$ is a fuzzy ideal of $\tilde{Y}$. As $\mu\tilde{f}(X) - \max \{[\mu\tilde{U}_\lambda: \mu\tilde{\lambda} \leq \mu\tilde{\lambda}^*]\} < \mu\tilde{f}(X) - \mu\tilde{f}(\max \{[\mu\tilde{f}^{-1}(\tilde{U}_\lambda): \mu\tilde{\lambda} \leq \mu\tilde{\lambda}^*]\})$, so $\mu\tilde{f}(X) - \max \{[\mu\tilde{U}_\lambda: \mu\tilde{\lambda} \leq \mu\tilde{\lambda}^*]\} \leq \tilde{f}(\tilde{I})$ implying thereby that fuzzy continuous image of fuzzy $\tilde{I}$ -compact space is fuzzy $\tilde{I}$ -compact.

Theorem 3.4. Let $(\tilde{X}, \tilde{\tau}, \tilde{I})$ be any fuzzy ideal space and let $\tilde{A}$ be a fuzzy subset of $\tilde{X}$ such that for each fuzzy open set $\tilde{U}$ containing $\tilde{A}$ there is fuzzy $\tilde{I}$ -compact set $\tilde{B}$ with $\mu\tilde{A} < \mu\tilde{B} < \mu\tilde{U}$. Then $\tilde{A}$ is fuzzy $\tilde{I}$ -compact.

Proof. Let $\{\tilde{U}_\lambda: \mu\tilde{\lambda} \leq \mu\tilde{\lambda}^*\}$ be a fuzzy $\tilde{\tau}_A$ -open cover of $\tilde{A}$ where $\mu\tilde{U}_\lambda = \min \{\mu\tilde{V}_\lambda, \tilde{A}\}$ such that $\tilde{V}_\lambda$ is fuzzy open in $\tilde{X}$. By the given condition, there exists a fuzzy $\tilde{I}$ -compact subset $\tilde{B}$ of $\tilde{X}$ such that $\mu\tilde{A} < \mu\tilde{B} < \cup \mu\tilde{V}_\lambda$. Then $\min \{\mu\tilde{V}_\lambda, \mu\tilde{B}: \mu\tilde{\lambda} \leq \mu\tilde{\lambda}^*\}$ is a fuzzy $\tilde{\tau}_B$ -open cover of $\tilde{B}$. As $\tilde{B}$ is fuzzy $\tilde{I}$ -compact, there exists a fuzzy finite subset $\tilde{\Lambda}^*$ of $\tilde{\Lambda}$ such that $\mu\tilde{B} - \max \{\min \{\mu\tilde{V}_\lambda, \mu\tilde{B}: \mu\tilde{\lambda} \leq \mu\tilde{\lambda}^*\}\} \leq \tilde{I}_B$. Let $\mu\tilde{B} - \max \{\min \{\mu\tilde{V}_\lambda, \mu\tilde{B}: \mu\tilde{\lambda} \leq \mu\tilde{\lambda}^*\}\} = \min \{\mu\tilde{I}_1, \mu\tilde{B}\}$. Here $\min \{\mu\tilde{I}_1, \mu\tilde{B}\} \leq \mu\tilde{I}_B$, where $\mu\tilde{I}_1 \leq \mu\tilde{I}$. Since $\mu\tilde{B} = \max \{\max \{\min \{\mu\tilde{V}_\lambda, \mu\tilde{B}: \mu\tilde{\lambda} \leq \mu\tilde{\lambda}^*\}\}, \min \{\mu\tilde{I}_1, \mu\tilde{B}\}\}$. Then $\min \{\mu\tilde{B}, \mu\tilde{A}\} = \min \{\max \{\max \{\min \{\mu\tilde{V}_\lambda, \mu\tilde{B}: \mu\tilde{\lambda} \leq \mu\tilde{\lambda}^*\}\}, \min \{\mu\tilde{I}_1, \mu\tilde{B}\}\}, \mu\tilde{A}\} \Rightarrow \mu\tilde{A} = \max \{\max \{\min \{\mu\tilde{V}_\lambda, \mu\tilde{B}: \mu\tilde{\lambda} \leq \mu\tilde{\lambda}^*\}\}, \min \{\mu\tilde{I}_1, \mu\tilde{B}, \mu\tilde{A}\}\} \Rightarrow \mu\tilde{A} = \max \{\max \{\min \{\mu\tilde{V}_\lambda, \mu\tilde{A}: \mu\tilde{\lambda} \leq \mu\tilde{\lambda}^*\}\}, \min \{\mu\tilde{I}_1, \mu\tilde{A}\}\} \Rightarrow \mu\tilde{A} - \max \{\min \{\mu\tilde{V}_\lambda, \mu\tilde{A}: \mu\tilde{\lambda} \leq \mu\tilde{\lambda}^*\}\} = \min \{\mu\tilde{I}_1, \mu\tilde{A}\} \leq \mu\tilde{I}_A$ implying thereby that $\tilde{A}$ is fuzzy $\tilde{I}$ -compact.

Corollary 3.4. If every fuzzy open subset of $\tilde{X}$ is fuzzy $\tilde{I}$ -compact, then every fuzzy subset of $\tilde{X}$ contained in fuzzy open subset is fuzzy $\tilde{I}$ -compact.

Theorem 3.5. If $\tilde{A}$ and $\tilde{B}$ are fuzzy $\tilde{I}$ -compact in fuzzy ideal space $(\tilde{X}, \tilde{\tau}, \tilde{I})$, then $\max \{\mu\tilde{A}, \mu\tilde{B}\}$ is fuzzy $\tilde{I}$ -compact in $\tilde{X}$.

Proof. Let $\{\tilde{U}_\lambda: \mu\tilde{\lambda} \leq \mu\tilde{\lambda}^*\}$ be an fuzzy open cover of $\max \{\mu\tilde{A}, \mu\tilde{B}\}$ in $\tilde{X}$. Then $\{\tilde{U}_\lambda: \mu\tilde{\lambda} \leq \mu\tilde{\lambda}^*\}$ is an fuzzy open cover of $\tilde{A}$ and $\tilde{B}$. Given $\tilde{A}$ and $\tilde{B}$ are fuzzy $\tilde{I}$ -compact, there exists $\mu\tilde{I}_1, \mu\tilde{I}_2 \leq \tilde{I}$ and fuzzy finite subset $\tilde{\Lambda}^*$ and $\tilde{\Lambda}_1$ of $\tilde{\Lambda}$ such that $\mu\tilde{A} - \max \{\mu\tilde{U}_{\lambda_i}: \mu\tilde{\lambda}_i \leq \mu\tilde{\lambda}^*\} = \tilde{I}_1$ and $\mu\tilde{B} - \max \{\mu\tilde{U}_{\lambda_k}: \mu\tilde{\lambda}_k \leq \mu\tilde{\lambda}^*\} = \tilde{I}_2$. $\mu\tilde{A} = \max \{\max \{\mu\tilde{U}_{\lambda_i}: \mu\tilde{\lambda}_i \leq \mu\tilde{\lambda}^*\}, \mu\tilde{I}_1\}$ and $\mu\tilde{B} = \max \{\max \{\mu\tilde{U}_{\lambda_k}: \mu\tilde{\lambda}_k \leq \mu\tilde{\lambda}^*\}, \mu\tilde{I}_2\}$. Now, $\max \{\mu\tilde{A}, \mu\tilde{B}\} = \max \{\max \{\max \{\mu\tilde{U}_{\lambda_i}: \mu\tilde{\lambda}_i \leq \mu\tilde{\lambda}^*\}, \mu\tilde{I}_1\}, \max \{\max \{\mu\tilde{U}_{\lambda_k}: \mu\tilde{\lambda}_k \leq \mu\tilde{\lambda}^*\}, \mu\tilde{I}_2\}\}$. $\max \{\mu\tilde{A}, \mu\tilde{B}\} = \max \{\max \{\mu\tilde{U}_{\lambda_i}, \mu\tilde{U}_{\lambda_k}\}: \mu\tilde{\lambda}_i \leq \mu\tilde{\lambda}^*, \mu\tilde{\lambda}_k \leq \mu\tilde{\lambda}^*\}, \max \{\mu\tilde{I}_1, \mu\tilde{I}_2\}$. This implies $\max \{\mu\tilde{A}, \mu\tilde{B}\} = \max \{\max \{\mu\tilde{U}_{\lambda_i}, \mu\tilde{U}_{\lambda_k}\}: \mu\tilde{\lambda}_i \leq \mu\tilde{\lambda}^*, \mu\tilde{\lambda}_k \leq \mu\tilde{\lambda}^*\}, \mu\tilde{I}$ where $\max \{\mu\tilde{I}_1, \mu\tilde{I}_2\} = \mu\tilde{I}(\max \{\mu\tilde{A}, \mu\tilde{B}\} - \max \{\max \{\mu\tilde{U}_{\lambda_i}, \mu\tilde{U}_{\lambda_k}\}: \mu\tilde{\lambda}_i \leq \mu\tilde{\lambda}^*, \mu\tilde{\lambda}_k \leq \mu\tilde{\lambda}^*\}) \leq \mu\tilde{I}$ implying thereby that is fuzzy $\tilde{I}$ -compact in $\tilde{X}$.

Corollary 3.5. Fuzzy Finite union of fuzzy $\tilde{I}$ -compact space $\tilde{X}$ is fuzzy $\tilde{I}$ -compact.

Theorem 3.6. The following are equivalent for a space $(\tilde{X}, \tilde{\tau}, \tilde{I})$

(a) $(\tilde{X}, \tilde{\tau}, \tilde{I})$ is fuzzy $\tilde{I}$ -compact.

(b) $(\tilde{X}, \tilde{\tau}^*, \tilde{I})$ is fuzzy $\tilde{I}$ -compact.

(c) For any family $\{\mu\tilde{F}_\lambda: \mu\tilde{\lambda} \leq \mu\tilde{\lambda}^*\}$ of fuzzy closed sets of $\tilde{X}$ such that $\min \{\mu\tilde{F}_\lambda: \mu\tilde{\lambda} \leq \mu\tilde{\lambda}^*\} = \tilde{\varphi}$, there exists a fuzzy finite subset $\tilde{\Lambda}^*$ of $\tilde{\Lambda}$ such that $\min \{\mu\tilde{F}_\lambda: \mu\tilde{\lambda} \leq \mu\tilde{\lambda}^*\} \leq \mu\tilde{I}$.

Proof. (a) $\Rightarrow$ (b) Let $\{\tilde{U}_\lambda: \mu\tilde{\lambda} \leq \mu\tilde{\lambda}^*\}$ be a fuzzy $\tilde{\tau}^*$ -open cover of $\tilde{X}$ such that $\mu\tilde{U}_\lambda = \mu\tilde{V}_\lambda - \mu\tilde{E}_\lambda$, where $\tilde{V}_\lambda$ fuzzy open in $\tilde{X}$ and $\mu\tilde{E}_\lambda \leq \mu\tilde{I}$. Now $\{\tilde{V}_\lambda: \mu\tilde{\lambda} \leq \mu\tilde{\lambda}^*\}$ is an fuzzy open cover of $\tilde{X}$ and hence there exists a fuzzy finite subset $\tilde{\Lambda}^*$ of $\tilde{\Lambda}$ such that $\mu\tilde{X} -$

$\max \{\tilde{V}_\lambda: \mu\tilde{\lambda} \leq \mu\tilde{\lambda}^*\} \leq \tilde{I}$. This implies that $\mu\tilde{X} - \max \{\tilde{U}_\lambda: \mu\tilde{\lambda} \leq \mu\tilde{\lambda}^*\} < (\max \{\mu\tilde{X} - \max \{\tilde{V}_\lambda: \mu\tilde{\lambda} \leq \mu\tilde{\lambda}^*\}\}, \max \{\tilde{E}_\lambda: \mu\tilde{\lambda} \leq \mu\tilde{\lambda}^*\} \leq \tilde{I}$. Therefore, $(\tilde{X}, \tilde{\tau}^*, \tilde{I})$ is fuzzy $\tilde{I}$ -compact.

(b) $\Rightarrow$ (a) It follows from $\tilde{\tau} < \tilde{\tau}^*$.

(a) $\Rightarrow$ (c) Let $\{\tilde{F}_\lambda: \mu\tilde{\lambda} \leq \mu\tilde{\lambda}\}$ be a family of fuzzy closed sets of $\tilde{X}$ such that $\min \{\tilde{F}_\lambda: \mu\tilde{\lambda} \leq \mu\tilde{\lambda}\} = \tilde{\varphi}$.

Then $\{\mu\tilde{X} - \mu\tilde{F}_\lambda: \mu\tilde{\lambda} \leq \mu\tilde{\lambda}\}$ is an fuzzy open cover of $\tilde{X}$. By (a) since $(\tilde{X}, \tilde{\tau}, \tilde{I})$ is fuzzy $\tilde{I}$ -compact, there exists a fuzzy finite subset $\tilde{\Lambda}^*$ of $\tilde{\Lambda}$ such that $\mu\tilde{X} - \max \{\mu\tilde{X} - \mu\tilde{F}_\lambda: \mu\tilde{\lambda} \leq \mu\tilde{\lambda}^*\} \leq \tilde{I}$. This implies that $\min \{\tilde{F}_\lambda: \mu\tilde{\lambda} \leq \mu\tilde{\lambda}^*\} \leq \tilde{I}$.

(c) $\Rightarrow$ (a) Let $\{\tilde{U}_\lambda: \mu\tilde{\lambda} \leq \mu\tilde{\lambda}\}$ be an fuzzy open cover of $\tilde{X}$, then $\{\mu\tilde{X} - \mu\tilde{U}_\lambda: \mu\tilde{\lambda} \leq \mu\tilde{\lambda}\}$ is a fuzzy collection of fuzzy closed sets and $\min \{\mu\tilde{X} - \mu\tilde{U}_\lambda: \mu\tilde{\lambda} \leq \mu\tilde{\lambda}\} = \tilde{\varphi}$. Hence there exists a fuzzy finite subset $\tilde{\Lambda}^*$ of $\tilde{\Lambda}$ such that $\min \{\mu\tilde{X} - \mu\tilde{U}_\lambda: \mu\tilde{\lambda} \leq \mu\tilde{\lambda}^*\} \leq \tilde{I}$. This implies that $\mu\tilde{X} - \max \{\tilde{U}_\lambda: \mu\tilde{\lambda} \leq \mu\tilde{\lambda}^*\} \leq \tilde{I}$. This shows $(\tilde{X}, \tilde{\tau}, \tilde{I})$ is fuzzy $\tilde{I}$ -compact.

REFERENCES

- [1] C. L. Chang, *Fuzzy Topological Spaces*, Journal of Mathematical Analysis and Applications **24** (1968), 182-190.
- [2] D. H. Foster, *Fuzzy Topological Groups*, Journal of Mathematical Analysis and Applications **67** (1979), 549-564.
- [3] J. A. Goguen, L. A. Zadeh. *Fuzzy sets. Information and control*, vol. 8 (1965), pp. 338–353. - L. A. Zadeh. *Similarity relations and fuzzy orderings. Information sciences*, vol. 3 (1971), pp. 177–200, Journal of Symbolic Logic **38** (1973), 656-657.
- [4] L. A. Gusev, I. M. Smirnova, *Fuzzy Sets: Theory and Applications (A Survey)*, Avtomatika i Telemekhanika **5** (1973), 66-85.
- [5] R. Lowen, *Fuzzy Topological Spaces and Fuzzy Compactness*, Journal of Mathematical Analysis and Applications **56** (1976), 621-633.
- [6] C. V. Negoitǎ, D. A. Ralescu, *Applications of Fuzzy Sets to Systems Analysis*, Springer, 1985.
- [7] M. Sarkar, *On Fuzzy Topological Spaces*, Journal of Mathematical Analysis and Applications **79** (1981), 384-394.