

THE UTILIZATION OF THE EMAD-FALIH INTEGRAL TRANSFORMATION TO SOLVE SOME CARDIOVASCULAR MODELS

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Abstract. The medical sector is one of the many scientific domains that may benefit from the use of mathematical models and algorithms to carry out a variety of activities that simplify and improve their job. The models that are utilized in the majority of medical areas are ones that can be anticipated or shaped using mathematical equations. These equations inspire mathematicians to handle them and alter them in order to discover the most accurate and direct form to answer the problems they provide. It's possible that some models of cardiovascular disease have solutions that are more complicated than algebraic models. Therefore, the use of integral transformations in the process of identifying the solutions to the aforementioned equations is quite beneficial. The Emad-Falih integral transformation is used throughout this article to do an analysis of the exact solutions to both of the cardiovascular models being discussed. For the purpose of calculating solutions to ordinary differential equations, the integral transform under study is a technique that is not only practical but also trustworthy and user-friendly.

INTRODUCTION

Emad A. K. and Sara F. M. introduced the Emad-Falih transformation in their published work, highlighting its strong correlation with the Laplace transformation and its ability to facilitate the identification of multiple properties and theorems [2, 9]. Subsequent to its initial development, a multitude of scholars, among them Emad A. and Sara F., conducted investigations into various implementations [3-6] of said integral transformation.

The medical field is an area of active research where mathematicians have invested significant efforts in applying various mathematical techniques. Integral transforms are particularly useful in efficiently handling diverse problems and, as such, have been extensively utilized to maximize their benefits. The present study endeavors to present the Emad-Falih transformation as a technique that simplifies the resolution of ordinary differential equations in the context of two distinct Cardiovascular models. Specifically, one of these models pertains to the monitoring of glucose concentration in the bloodstream during continuous intravenous glucose administration. The aorta experiences pressure, as indicated by sources [1, 7, 8, 10].

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BASIC CONCEPTS

The following section presents fundamental definitions and significant properties pertaining to the Emad-Falih integral transform.

Definition (2.1) [2]: considering A as a function set which is defined by:

$$A = \left\{ f(t) : \exists K, m_1, m_2 > 0, |f(t)| K e^{m_1^2 |t|} \text{ if } t \in (-1)^j \times [0, \infty), j = 1, 2 \right\}.$$

Where $f(t)$ is a function belongs to the set A , the finite constant number is K , m_1 and m_2 may or may not be finite.

Definition (2.2) [2]: The integral transformation indicated by $EF\{\cdot\}$ is the Emad-Falih transformation, it is defined on the set A as:

$$EF\{f(t)\} = T(\varphi) = \frac{1}{\varphi} \int_{t=0}^{\infty} e^{-\varphi^2 t} \cdot f(t) dt.$$

Where $t \geq 0$, $m_1 \leq \varphi \leq m_2$ and the variable φ is utilized as a coefficient for the variable t in the function f .

Theorem (2.3) [2]: (Linearity Property)

$$EF\{A f_1(t) \pm B f_2(t)\} = A EF\{f_1(t)\} \pm B EF\{f_2(t)\}.$$

A and B are constants, $f_1(t)$ and $f_2(t)$ are t functions.

Theorem (2.4) [2]: Let $T(\varphi)$ is the Emad-Falih transformation of $(EF\{f(t)\} = T(\varphi))$, then:

$$EF\left\{\frac{df}{dt}\right\} = \frac{-f(0)}{\varphi} + -\varphi^2 T(-\varphi). \quad (1)$$

And,

$$EF\left\{\frac{d^2 f}{dt^2}\right\} = \frac{-f'(0)}{\varphi} + \varphi^2 EF\left\{\frac{df}{dt}\right\}$$

In general: $EF\left\{\frac{d^n f}{dt^n}\right\} = \frac{-f^{(n-1)}(0)}{\varphi} + \varphi^2 EF\left\{\frac{d^{n-1} f}{dt^{n-1}}\right\}$, considering n as a positive integer.

Table (1) Presents the Emad-Falih integral transformations of basic functions.

Table 1. The application of the Emad-Falih to several fundamental functions [2].

$f(t)$	$EF\{f(t)\} = T(\varphi)$
$k \equiv \text{Constant}$	$\frac{k}{\varphi^3}.$
t^n where n is +ive integer number	$\frac{n!}{\varphi^{2n+3}}.$
$e^{at} \ a \equiv \text{constant}$	$\frac{1}{\varphi(\varphi^2 - a)}.$
$\sin(at)$	$\frac{a}{\varphi(\varphi^4 + a^2)}.$
$\cos(at)$	$\frac{\varphi}{(\varphi^4 + a^2)}.$
$\sinh(at)$	$\frac{a}{\varphi(\varphi^4 - a^2)}.$
$\cosh(at)$	$\frac{\varphi}{(\varphi^4 - a^2)}.$

APPLICATIONS

This section presents the implementation of the Emad-Falih transformation on two distinct cardiovascular models.

Application (3.1) (Blood Glucose Concentration)

During continuous intravenous glucose injection, the concentration of glucose in the blood is $G(t)$ exceeding the baseline value at the start of the infusion. The function $G(t)$ satisfies the initial value problem (I.V.P.), [1].

$$\left. \begin{aligned} G'(t) + kG(t) &= \frac{\alpha}{\gamma}, \text{ where } t > 0 \\ \text{and, } G(0) &= 0 \end{aligned} \right\} \quad (2)$$

The variables in this equation are k, α , and γ , which respectively represent the constant velocity of elimination, the rate of infusion, and the volume in which glucose is distributed.

The Emad-Falih transformation technique will be utilized to assess the concentration of glucose present in the bloodstream.

Upon bilateral application of the Emad-Falih integral transformation to equation (2), the resulting expression is obtained:

$$EF\{G'(t)\} + k EF\{G(t)\} = \frac{\alpha}{\gamma} EF\{1\} \quad (3)$$

Let $EF\{G(t)\} = T(\varphi)$. By utilizing the initial value problem (I.V.P.) and the integral transformation outlined in table (1), the equation (3) can be rearranged with the aid of equation (1) as:

$$\begin{aligned} -\frac{G(0)}{\varphi} + \varphi^2 T(\varphi) + k T(\varphi) &= \frac{\alpha}{\gamma} \cdot \frac{1}{\varphi^3}, \\ T(\varphi) &= \frac{\alpha}{\gamma - \varphi^3(\varphi^2 + k)} \end{aligned} \quad (4)$$

Upon rearranging equation (4) and dividing the fractions, the resulting expression is:

$$\begin{aligned} T(\varphi) &= \frac{\alpha}{\gamma \varphi \varphi^2(\varphi^2 + k)}, \\ &= \frac{\alpha}{\gamma \varphi} \left[\frac{1}{k\varphi^2} - \frac{1}{k(\varphi^2 + k)} \right], \\ &= \frac{\alpha}{\gamma} \left[\frac{1}{k\varphi^3} - \frac{1}{k\varphi(\varphi^2 + k)} \right], \\ &= \frac{\alpha}{\gamma k} \left[\frac{1}{\varphi^3} - \frac{1}{\varphi(\varphi^2 + k)} \right] \end{aligned}$$

After applying the inverse Emad-Falih transformation, the concentration of glucose present in the bloodstream can be obtained as follows:

$$G(t) = \frac{\alpha}{\gamma k} (1 - e^{-kt}).$$

Application (3.2): (Aorta Pressure)

The heart's contraction facilitates the transportation of blood into the aorta. The initial value problem denoted by Debnath and Bhatta [1] is concerned with the aortic pressure function $p(t)$ as:

$$\left. \begin{aligned} p'(t) + \frac{c}{k} p(t) &= cA \sin(wt), \\ p(0) &= p_0. \end{aligned} \right\} \quad (5)$$

c, k, A and p_0 are constants. The Emad-Falih transformation technique is utilized to derive the pressure in the aorta.

With the bilateral application of the Emad-Falih integral transformation to equation (5), the resulting expression is:

$$EF\left\{\frac{dp(t)}{dt}\right\} + \frac{c}{k} EF\{p(t)\} = cA EF\{\sin(wt)\} \quad (6)$$

Let $EF\{p(t)\} = T(\varphi)$, by applying the initial value problem and utilizing the transformations outlined in Table 1, the rearrangement of Equation (6) can be

expressed as a function of Equation (1) as:

$$\begin{aligned}\frac{-p(0)}{\varphi} + \varphi^2 T(\varphi) + \frac{c}{k} T(\varphi) &= \frac{cAw}{\varphi(\varphi^4 + w^2)}, \\ \left(\varphi^2 + \frac{c}{k}\right) T(\varphi) &= \frac{cAw}{\varphi(\varphi^4 + w^2)} + \frac{p_0}{\varphi}, \\ T(\varphi) &= \frac{cAw}{\varphi(\varphi^4 + w^2)\left(\varphi^2 + \frac{c}{k}\right)} + \frac{p_0}{\varphi\left(\varphi^2 + \frac{c}{k}\right)}.\end{aligned}$$

After performing basic calculations and applying partition fractions along with the inverse of the Emad-Falih transformation to the given expression, the resultant value obtained represents the amount of pressure in the aorta.

$$p(t) = p_0 e^{\frac{-c}{k}t} + \frac{cAwrk^2}{\omega^2 k^2 + c^2} \left(\frac{c}{\omega k} \sin(\omega t) - \cos(\omega t) + e^{\frac{-c}{k}t} \right).$$

CONCLUSION

This paper uses integral transformations, specifically the Emad-Falih integral transform, in the medical field. The benefit of this transformation, based on the Laplace transformation, comes from its ability to deal with some cardiovascular models. The Emad-Falih integral transformation has proven its applicability, reliability, effectiveness, and ease of application in two applications: finding the blood glucose concentration and finding the aortic pressure presented in this work.

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