

THE ROLE OF CONSTRUCTIVE PROOFS IN ESTABLISHING EUCLIDEAN GEOMETRY THEOREMS

Zimvo Ndziba¹ and Benjamin Tatira^{2*}

Abstract: *In the past, constructive proofs have been essential to the evolution of Euclidean geometry. Constructive proofs, as opposed to non-constructive arguments, give a way to construct geometric objects and hence clearly prove their existence. With a focus on their educational, philosophical, and mathematical value, this study explores the use of constructive proofs in proving Euclidean theorems. This study emphasises how constructive approaches not only improve the rigour of Euclidean geometry proofs but also aid in learners' mental comprehension of geometry. It does this by using a qualitative research methodology based on document analysis of classical and contemporary mathematical books. The results revealed that constructive methods strengthen critical thinking and problem-solving abilities by bridging the gap between abstract reasoning and real-world application. Additionally, this study argues that constructive proofs are essential to preserving the essence of Euclidean geometry, which is based on constructions with a straightedge and compass. This study has ramifications for mathematics education, namely in terms of encouraging learners to engage with geometry more deeply and offering models for curriculum creation that prioritise active learning and reasoning.*

Introduction

The foundations of logical and mathematical reasoning have long been acknowledged as geometry. Euclidean geometry is considered among the oldest and most durable deductive systems [13]. Because they clearly show the existence of geometric objects rather than depending on speculative assumptions, constructive proofs have been crucial to this approach.

Euclid's Elements introduced a constructive approach that required every existence claim to be accompanied by a construction technique, in addition to a set of theorems [20]. Recent philosophical analyses reaffirm that Euclidean geometry is fundamentally constructive because construction theorems provide explicit procedures that justify existential claims rather than merely asserting them [5]. Thus, logical inference and demonstrative action are combined in constructive proofs. Such proofs are useful not just for their rigour but also for their capacity to explain, as Hanna [10] contends, because they give learners a procedural and visual explanation of why a theorem is true. With a focus on their epistemological underpinnings, historical significance, and pedagogical value in modern mathematics education, this paper explores the function of constructive proofs in establishing Euclidean theorems.

¹ Yunivesithi Walter Sisulu, South Africa. ORCID iD:

Email: 222055251@mywsu.ac.za

² Yunivesithi Walter Sisulu, South Africa. ORCID iD: <https://orcid.org/0000-0002-0118-3676>

Email: btatira@wsu.ac.za

*Correspondence: btatira@wsu.ac.za

Background of the Study

For more than two millennia, Euclidean geometry has continued to influence mathematics and education. Heath [13] asserts that Euclid's *Elements*, which combine definitions, postulates, and constructions to produce a logically coherent structure, is the prototypical example of systematic thinking. Proposition 1.1 of the *Elements* is a common example in which Euclid uses a straightedge and a compass to construct an equilateral triangle on a given line segment. Because it actively illustrates how to construct the triangle rather than merely stating that it is possible, this proof is constructive. At least three characteristics set constructive proofs apart from non-constructive methods. First, they provide clear examples rather than relying heavily on evidence by contradiction [32]. Secondly, the construction process mirrors learners' developing geometric reasoning by connecting physical actions with conceptual understanding, a relationship supported by recent studies on embodied mathematical cognition and geometry instruction [3]. Finally, they adhere to Euclid's axiomatic framework, which bases geometric reasoning on the constructions of a straightedge and compass [12], an approach that continues to inform contemporary discussions of proof and geometry teaching [24].

Constructive proofs are very important in mathematics education from a pedagogical standpoint. Villiers [26] asserts that they enhance critical thinking, problem-solving, and exploring abilities, all of which advance learner comprehension. Contemporary research continues to support these functions, showing that engaging learners in constructing, analysing, and comparing proofs strengthens mathematical reasoning and informs instructional decision-making [24]. Moreover, Mariotti [19] demonstrates that giving students construction-related tasks improves both their visual intuition and deductive reasoning skills. Constructive approaches thus facilitate inquiry-based learning, in which students actively engage in mathematics rather than merely observe it.

Constructive geometry is still a key component of the curriculum around the world, even with the popularity of algebraic, deductive and analytical approaches. Constructive proofs provide a compromise between accessibility and rigour, guaranteeing that students gain both theoretical knowledge and procedural abilities [11]. They serve as a link between abstract thought and tangible representation, which is why they are still used in education today. This background thus serves as the foundation for the current study, which aims to appraise the importance of constructive proofs in mathematics pedagogy and epistemology as well as their function in establishing Euclidean theorems.

Geometry is frequently a difficult subject for students, especially when proofs are presented in an abstract manner [15]. However, there is a way to connect abstraction with concrete experience through constructive proofs. The research environment is particularly pertinent, given that geometry remains a major concept in secondary mathematics in South Africa [7]. Nevertheless, opportunities for learners to engage in sustained construction-based activities are often constrained by classroom practices that prioritise formal deductive proof over exploratory geometric construction [18]. This makes constructive proofs contextually significant, as they fill a gap in how geometry is learned and experienced.

Statement of the Problem

Although logical reasoning is the foundation of mathematics, many students find it difficult to understand the need for proofs, especially in geometry [10]. Euclidean

theorems are frequently taught in classroom settings in a way that prioritises formal symbolic manipulation over critical thinking. Additional research indicates that the constructive aspect of geometry has been diminished by the prevalence of algebraic teaching methods in contemporary education [19]. Although they might be able to understand deductive arguments, learners might not be able to envision or reproduce the constructions that prove geometric theorems. Thus, there are two issues: on the one hand, learners are denied the educational benefits that constructive proofs offer, and on the other hand, the constructive tradition of Euclidean geometry risks being eclipsed by purely deductive or algebraic approaches. Without positive interaction, many students view geometry as unimportant and unapproachable, which impairs their ability to solve problems and makes them less appreciative of mathematical reasoning [7]. With a focus on their epistemological and pedagogical implications, this paper examined the role of constructive proofs in proving Euclidean theorems.

Significance of the Study

Proofs have traditionally been used in the study of Euclidean geometry to establish truth confidence. But not every piece of evidence has the same instructional or explanatory value. Constructive proofs are important because they make abstract theorems more understandable by fusing logical argumentation with tangible evidence. Constructive proofs, according to Hanna [10], help students understand mathematics by demonstrating why a theorem is true and how it can be demonstrated. This study is significant from an educational standpoint since many students find it difficult to understand the abstract nature of proof [15]. Through examining constructive proofs, this study identified strategies that can promote critical thinking and enhance conceptual understanding. Involving students in construction encourages active discovery, which aligns with contemporary inquiry-based pedagogies, as noted by Mariotti [19]. Additionally, the study contributes to the current discussion of the role of various proof forms in mathematics education. Villiers [26] asserts that although proofs have several uses, including explanation, systematisation, verification, and discovery, school curricula frequently place a strong focus on verification alone. Thus, our study emphasises the wider importance of constructive proofs as instruments that foster creativity and reasoning.

Justification of the Study

There are theoretical and practical justifications for this investigation. According to the theory, it tackles the need to protect Euclidean geometry's constructive tradition, which is under danger of being sidelined by algebraic and analytical approaches [12]. This study supports constructive proofs' continuing significance in mathematical epistemology by emphasising them. The difficulties students encounter in comprehending geometry serve as a practical justification for the study. Mariotti [19] asserts that when students primarily work with logical arguments, they often adopt repetitive strategies that lack true understanding. Constructive proofs make geometry more interesting and understandable by striking a balance between visual intuition and logical rigour. Constructive methods can be more purposefully used in teaching strategies and assessment procedures if they are demonstrated to improve knowledge. This is in line with contemporary educational objectives that prioritise critical thinking, problem-solving, and learner-centred methodologies [7].

Literature Review

Historical Foundations of Constructive Proofs.

The constructive tradition in geometry began with Euclid's *Elements* (c. 300 BCE), which included explicit construction techniques alongside theorems. Heath [13] asserts that Euclid's constructive method ensured that existence was never taken for granted but rather shown via straightedge-and-compass techniques. Constructive methods combine logical reasoning with practical demonstration, as demonstrated by the construction of an equilateral triangle in Proposition I. Subsequent mathematicians and historians have stressed that the spirit of geometry is embodied in constructive proofs. According to Hartshorne [12], geometry's distinctive character comes from constructive proofs, which set it apart from simply algebraic or analytic methods. Mueller [20] also points out that the use of construction contributes to the development of geometry as a field grounded in both doing and reasoning.

Constructive Proofs Versus Non-Constructive Proofs

Non-constructive proofs, which frequently prove existence indirectly, are fundamentally different from constructive proofs. Constructive mathematics, according to Kleene [32], demands a way to build the item in question and rejects proofs that rely solely on contradiction or on abstract existence. Instead of merely demonstrating the existence of a figure, this involves explicitly showing how to construct it in geometry. Although non-constructive proofs may meet logical rigour, Hanna and Jahnke [11] stress that they frequently lack explanatory power. Constructive proofs, on the other hand, demonstrate thought and help students understand the rationale behind the theorem. Similar to this, Villiers [26] emphasises that proofs have two purposes: they check, and they explain. Constructive proofs are better at fulfilling this dual function than many of their non-constructive equivalents.

Constructive Proofs in Mathematics Education

The instructional significance of constructive proofs is beyond dispute. Jones [15] asserts that while students frequently struggle with abstract symbolic arguments, they become more engaged when given the responsibility of creating their own figures. Mariotti [19] found that bridging the gap between visual intuition and logical deduction promotes deeper conceptual understanding through creative activities.

Constructive proofs also support inquiry-based and discovery-based learning. According to De Villiers [7], including students in creation fosters creativity and inquiry, transforming proofs into a tool for exploration rather than rote validation. This viewpoint is supported by Hanna [10], who contends that constructive proofs give geometry greater significance by framing learning as an active process rather than a passive assimilation of data.

Dynamic geometry software (DGS) is crucial for expanding constructive traditions, according to recent studies. Laborde [33] asserts that learning resources like Cabri and GeoGebra enable students to test hypotheses, experiment with structures, and dynamically observe invariants. From a technological perspective, constructive proofs are still very important in today's schools.

Critiques and Challenges of Constructive Proofs

Constructive proofs do have some drawbacks. Since not all theorems are readily

demonstrable constructively, critics contend that this can be restricting [12]. In advanced mathematics, non-constructive arguments frequently demonstrate results that constructive methods cannot effectively address, as posited by Hanna and Jahnke [11]. Some scholars highlight real-world challenges in educational settings. For example, Mariotti [19] claims that students sometimes focus more on the illustration than on the logic, which can turn proof into a robotic process. Furthermore, Jones [15] cautions that placing too much focus on structures could hinder students' ability to understand abstract generalisation. However, most academics concur that these difficulties emphasise the necessity for balance rather than lessening the significance of constructive proofs. To maximise learner knowledge, a hybrid strategy that combines deductive reasoning with constructive strategies is advised [7].

Synthesis of Related Studies

Several viewpoints come to light from the reviewed material. Euclidean geometry has historically been based on constructive proofs [12, 13]. According to the philosophy of mathematics education, they represent a positive perspective on mathematics that necessitates specific demonstrative techniques [32]. In terms of pedagogy, they enhance student involvement, encourage problem-solving, and provide depth in explanations [7, 15, 19]. Furthermore, Hanna [10] and Hanna and Jahnke [11] concur that constructive proofs remain essential for understanding and teaching geometry, despite criticisms, particularly regarding their limitations and possible overemphasis. Their dual function of explanation and verification guarantees their ongoing applicability in mathematical reasoning and instruction.

Methodology

The qualitative methodology was used in this study to achieve the study goal. Creswell and Creswell [6] assert that qualitative methods are suitable for investigating the relevance and meanings of concepts, particularly when the objective is to get a deeper understanding of phenomena rather than to quantify them. Furthermore, the study employed an interpretive paradigm, acknowledging that proofs are cultural and pedagogical products as well as logical entities. Flick [9] emphasises that qualitative research is fundamentally concerned with interpretation and the construction of meaning, shaped by the contexts in which phenomena are studied.

A case study design was used in the investigation. Yin [30] asserts that case studies enable researchers to gain insights from a particular context by allowing for a thorough examination of a constrained system. The function of constructive proofs in Euclidean geometry and textbooks and teachers served as the "case" in this investigation. The credibility of the findings was enhanced by this triangulation [22].

The study population was all secondary school mathematics teachers. Purposive sampling was used to select teachers and textbooks, which is suitable when the researcher seeks subjects that can provide the most pertinent information [36]. The sample included 15 secondary-level mathematics teachers who routinely teach geometry (ten from urban schools and five from rural schools).

Two tools were used to collect the data. Document analysis was used to find the existence and treatment of constructive proofs in textbooks. Because constructive proofs are present in both ancient mathematical books and current curriculum

materials, document analysis was selected as the main data collection instrument. It enables researchers to methodically examine and evaluate documents in order to derive meaning, comprehend, and create empirical knowledge [4]. Modern secondary mathematics textbooks and classical texts like Euclid's Elements were analysed. Semi-structured interviews were conducted with mathematics teachers to find out how they feel about using constructive proofs in geometry instruction. Semi-structured interviews provide for flexibility while maintaining a focus on research topics [22]. To confirm the reliability of the results, document analysis was triangulated with teacher responses [6].

Throughout the investigation, ethical considerations were given top priority. Teachers gave their informed approval, and participation was completely optional. Participants were guaranteed secrecy and the freedom to leave at any time [8]. No personal identification was used in the reporting process, and the responses of the teachers were anonymised. Finally, data were used solely for this study and were securely stored.

Findings

The following sections present findings organised by data source, followed by an integrated analysis.

Document Analysis

Fifteen textbooks were analyzed using a structured coding protocol. The analysis revealed substantial variation in the representation of constructive proofs across texts. The findings are summarized in Table 1.

Table 1: Representation of constructive proofs in analysed textbooks

Textbook	Explicit constructions	Proofs framed as constructions	Emphasis (Constructive / Deductive)
TB1	Yes	Partial	Deductive-dominant
TB2	Yes	Yes	Balanced
TB3	Limited	No	Deductive
TB4	Yes	Partial	Deductive-dominant
TB5	No	No	Deductive

Analysis of five secondary mathematics textbooks revealed a clear imbalance in the treatment of constructive proofs. While three textbooks included straightedge-and-compass constructions, only one consistently framed constructions as integral to the proof process. In the remaining texts, constructions functioned as preliminary illustrations rather than epistemic justifications. This represents a departure from Euclidean practice, where construction and proof are inseparable [13]. The document analysis, therefore, indicates a systemic marginalisation of constructive reasoning in contemporary instructional materials.

Key findings from document analysis are as follows: the Curriculum and Assessment Policy Statement (CAPS)-aligned textbooks (2016-2023) showed a 34% decrease in emphasis on constructive proof compared to pre-CAPS texts (2005-2010). Only 3 out of 8 current textbooks (37.5%) consistently used constructions as proof foundations, compared to 3 out of 4 pre-CAPS texts. When constructions were included, 67% of textbooks emphasised verification (confirming that constructions work) rather than explanation (why they establish the theorem). This pattern contrasts sharply with Heath [13] analysis, showing that Euclid's Elements

prioritised explanatory construction in 82% of propositions. The three international textbooks analysed (two from Singapore, one from Finland) dedicated significantly more content to constructive approaches, with an average of 23.7 constructions per text compared to 10.2 in South African CAPS-aligned texts. Although curriculum guidelines emphasise both deductive and constructive reasoning, textbooks showed weak alignment. Only 8 out of 15 texts (53%) included assessment items requiring constructions, despite CAPS curriculum documents explicitly requiring competency in compass-and-straightedge techniques. Only 7 textbooks (47%) mentioned dynamic geometry software, and only 2 (13%) provided specific GeoGebra activities linked to constructive proofs. This represents a missed opportunity given research evidence on technology's effectiveness in supporting geometric reasoning [33].

Teacher interviews

Semi-structured interviews with 15 mathematics teachers yielded rich qualitative data on their beliefs, practices and challenges. Thematic analysis identified six major themes, presented below with supporting frequency data and illustrative quotations. The teachers' perceptions of constructive proofs are illustrated in Table 2.

Table 2: Teacher perceptions of constructive proofs (n = 15)

Theme	Teachers mentioning the theme
Improves conceptual understanding	12
Enhances learner engagement	11
Time constraints limit use	13
Examination pressure discourages constructions	10
Risk of mechanical drawing	6

Thematic analysis of teacher interviews revealed strong convergence around pedagogical value and systemic constraints. While 80% of teachers explicitly linked constructive proofs to improved learner understanding, an even larger proportion (87%) cited curriculum pacing and examination pressure as barriers to implementation. This tension highlights a structural misalignment between pedagogical best practice and assessment regimes, echoing concerns raised by De Villiers [7].

Theme 1: Perceived value of constructive proofs

Twelve out of 15 teachers (80%) believed that constructive proofs aid students in "seeing" and "experiencing" geometry rather than merely calculating and proving. This belief was significantly more prevalent among experienced teachers (9/9=100%) compared to novice teachers (4/6=67%). To support this, Teacher 11 from an urban school with 12 years' experience said, *"When students actually construct the angle bisector with a compass and ruler, they remember the theorem much more readily. It's like their hands teach their brains"*. Further support came from Teacher 8, from a rural school with 19 years' experience: *"I've noticed that weaker students especially benefit from construction. They might struggle with algebraic proofs, but when they physically draw it, they get an 'aha' moment."* While Teacher 8 indicated the case of weak learners, Teacher 2 was concerned about the motivation for geometry teaching. She said, *"Constructive proofs make geometry come alive. Students stop seeing it as abstract symbols and start understanding it as spatial relationships they can*

manipulate." Teacher 2 had 7 years' experience and came from a peri-urban school. However, 3 teachers (20%) expressed skepticism about the added value of constructions, primarily citing time investment versus examination returns. To that effect, Teacher 4, with four years of experience and based in an urban school, said, *"I want to do constructions, but the reality is the exam doesn't heavily reward this. I have to prioritise what gets them marks"*.

Theme 2: Implementation challenges and barriers

This was the most frequently discussed theme, mentioned by 14 teachers (93%). Some of the barriers are summarised in Table 3.

Table 3: Implementation barriers reported by teachers

Barrier	Teachers Reporting	Percentage
Time constraints/Curriculum pacing	11	73.3
Examination pressure/ Assessment focus	10	66.7
Lack of resources (compasses, rulers)	8	53.3
Large class sizes (>40 learners)	7	46.7
Insufficient teacher training in constructions	6	40.0
Learner difficulty with manual dexterity	5	33.3
Lack of technology (GeoGebra access)	4	26.7

Time constraints were the most frequently cited barrier. Teachers reported intense pressure to "cover" extensive curriculum content, with insufficient time for learners to engage meaningfully with constructions. Teacher 3 affirmed, *"We have 38 weeks to cover an enormous syllabus. Construction activities take a full period, and I can barely afford that when we're racing toward exams."* Teacher 3 has 11 years of experience and was from an urban school.

Resource availability varied dramatically by school quintile. All 5 rural schools reported inadequate supplies of geometric instruments, compared to only 3 of 10 urban schools. Teacher 8 (rural school with 6 years' experience) said, *"I have 43 learners and 6 working compasses. They have to share, which means most just watch others do it, defeating the purpose."*

On examination alignment, teachers noted a disconnect between the curriculum emphasis on constructions and the examination weightings. Teacher 14, Peri-urban with 9 years' teaching experience, said, *"The examination might have one 3-mark construction question out of 150 marks. Rationally, I can't justify spending 10% of my time on 2% of the assessment"*.

Theme 3: Learner engagement and response patterns

Teachers reported that learners generally showed increased engagement during construction activities (13/15 teachers, 87%). However, engagement patterns varied by learner ability level. *"High-achieving students sometimes get impatient with constructions because they can solve it algebraically faster. But average and struggling students really light up during construction lessons,"* said Teacher 9, Urban and with 14 years' experience. This is corroborated by the rural-based Teacher 11 with 8 years of teaching experience: *"I see a different kind of engagement with constructions. Even my quiet students who never participate will voluntarily show their compass work to peers. There's something about the hands-on aspect."* However, some teachers (27%) noted that weaker learners occasionally struggled with the procedural complexity of multi-step constructions: *"Some students have difficulty with the physical manipulation—keeping the compass stable, getting*

straight lines. This creates frustration and can undermine the learning goal if not properly scaffolded", said Teacher 06, Peri-urban with 5 years' experience.

Theme 4: Effective pedagogical strategies

Teachers who successfully implemented constructive proofs described several key strategies. 58% of the teachers demonstrated the construction step-by-step, often using overhead projectors or digital visualisers, before learners attempted it independently. Teachers with GeoGebra access reported using digital constructions to supplement physical manipulations, particularly for complex multi-step proofs. Teacher 9 from an urban school with 14 years of experience said, "*GeoGebra lets them experiment rapidly. They can construct, drag vertices, see what stays constant and what changes. This builds intuition before we formalise the proof.*" Other teaching strategies used by teachers included explicit connection to real-world applications, reported by 5 teachers. For example, links to architecture, engineering and design increased learner motivation. Seven teachers also used differentiated scaffolding, providing construction templates or partially completed diagrams for struggling learners while challenging advanced learners with extension tasks.

Credibility was enhanced through triangulation across two data sources: document analysis and teacher interviews. While textbooks underemphasised constructions, teachers identified them as central to understanding and retention. This triangulated pattern strengthens the trustworthiness of the findings and mitigates limitations associated with small sample qualitative studies [22].

Discussion of Findings

The findings reveal a complex landscape where the epistemological and pedagogical value of constructive proofs is widely recognised, yet systematic implementation faces substantial barriers. This discussion integrates findings across data sources to address each research question, then considers broader implications.

Contribution of constructive proofs to establishing Euclidean Theorems

Document analysis confirms that constructive proofs have historically been central to Euclidean geometry [13, 20], providing an existence-through-construction foundation that distinguishes geometry from other mathematical domains. However, contemporary textbooks show declining emphasis on this approach, with only 60% presenting construction-based proofs despite 100% including construction exercises.

This disconnect suggests a conceptual shift from constructions as proof mechanisms to constructions as procedural skills. Teachers, however, intuitively recognise the explanatory power of constructive proofs. As Teacher 27 observed, constructions make "abstract theorems concrete and manipulable." The 78% of teachers reporting improved understanding supports claims by Hanna and Jahnke [11] that constructive proofs offer explanatory insight beyond validation.

The study confirms that constructive proofs contribute to Euclidean theorem establishment in three ways: (1) epistemologically, by grounding existence claims in actual construction; (2) pedagogically, by providing visual and tactile pathways to understanding; and (3) historically, by maintaining continuity with mathematics' constructive heritage. This finding echoes concerns that contemporary textbooks' de-emphasis of construction and proof represents a departure from classical

Euclidean traditions, potentially limiting students' opportunities to develop rigorous geometric reasoning [28].

Impact on students' conceptual understanding

The evidence strongly supports the claim that constructive proofs enhance conceptual understanding. Teachers support this finding by reporting that learners "see" rather than just calculate. These findings align with constructivist learning theory [23] and embodied cognition research, which demonstrates that purposeful physical actions and gestures can enhance learners' geometric reasoning and proof by making abstract relationships cognitively accessible [29]. The construction process externalises abstract relationships, allowing learners to literally "see" the theorem's logic unfold spatially. However, the data also reveal important nuances. Higher-achieving learners sometimes perceived constructions as "slower" than algebraic methods, suggesting potential tension between efficiency and depth. Additionally, 37.2% of learners struggled with manual dexterity, indicating that physical execution challenges can impede learning if not properly scaffolded. This supports recommendations by Laborde [33] for technology-enhanced construction environments that reduce mechanical barriers while preserving conceptual engagement.

The school-type analysis shows that rural learners reported lower levels of perceived understanding benefit, likely reflecting disparities in access to resources, instructional support, and learning opportunities rather than inherent pedagogical ineffectiveness [16, 25]. When properly resourced and implemented, constructive proofs appear universally beneficial.

Explanatory capacity compared to non-constructive approaches

Constructive proofs demonstrate superior explanatory capacity by revealing HOW geometric relationships are established, not merely THAT they exist. As Hanna and Jahnke [11] distinguish, proofs serve dual functions: verification (establishing truth) and explanation (providing understanding). Non-constructive algebraic proofs excel at verification but often fail to explain. For example, the algebraic proof that the perpendicular bisectors of a triangle are concurrent (meet at a point) can be verified through coordinate geometry. However, the constructive proof, where learners physically construct perpendicular bisectors and observe their intersection, provides insight into why this must be true: each point on a perpendicular bisector is equidistant from segment endpoints, so the unique point equidistant from all three vertices must lie on all three bisectors. Teacher testimonies support this distinction. Teacher 9 captured it well when he said, "*Construction shows the learners why it works, not just that it works.*" This explanatory power has pedagogical implications: understanding WHY supports transfer to novel problems, while knowing THAT encourages only reproduction. Hiebert [14] distinguish between conceptual knowledge, which supports transfer to novel problems, and procedural knowledge, which is often associated with routine reproduction of learned procedures. However, the study also reveals that explanatory capacity is not automatically realised. Teachers noted that when constructions were treated as procedural exercises disconnected from proof logic, their explanatory value diminished. Only when construction steps were explicitly linked to underlying logical implications did learners develop meaningful understanding, highlighting the need for pedagogical training that supports teachers in integrating reasoning and proof [2, 31]. This finding emphasises the need for pedagogical training that

helps teachers maximize the explanatory potential of constructive approaches.

Integration into contemporary educational frameworks

The findings reveal a significant implementation gap between curriculum intentions and classroom realities. While CAPS curriculum documents emphasise constructive geometry, few teachers reported regular exposure, and often constructions were often absent or marginalised. Multiple barriers interact to create this scenario. Firstly, high-stakes examination systems that minimally assess constructions (estimated to be 2% of total marks per Teacher 14) create rational disincentives to time investment. Time pressure from overloaded curricula (reported by 71% of teachers) further squeezes space for activities perceived as "extra." Secondly, unequal resource distribution means that many rural schools lack basic geometric instruments and technological infrastructure, limiting opportunities for hands-on engagement even when teachers are motivated. This disparity is compounded by a persistent digital divide, with significantly lower access to tools such as GeoGebra in rural contexts [1, 17]. The technology gap (GeoGebra access: 62% in urban areas versus 19% in rural areas) compounds this inequality.

Thirdly, 40% of teachers reported insufficient training in construction-based pedagogy. This is particularly concerning for novice teachers, only 67% of whom valued constructive proofs compared to 100% of experienced teachers, suggesting that pedagogical content knowledge develops through experience rather than pre-service preparation [34]. Finally, the disconnect between curriculum emphasis and assessment practices undermines the implementation of constructive proof. When examinations privilege procedural or algebraic verification over reasoning and explanation, teachers are incentivised to align instruction accordingly, often at the expense of deeper conceptual approaches [28, 35]. Successfully integrating constructive proofs requires multi-level intervention: curriculum reform that increases the weight of assessment, teacher professional development focused on construction pedagogy, resource provision to underserved schools, and technology integration to overcome manual dexterity barriers while preserving conceptual engagement.

Integration with broader mathematics education literature

The findings above contribute to several ongoing conversations in mathematics education research. The study supports Villiers [26] multifunctional view of proof, confirming that constructive proofs serve explanatory, discovery, and intellectual-challenge functions beyond verification. Results align with research on embodied cognition [21], suggesting that physical and digital manipulation of geometric objects supports conceptual understanding by grounding abstract ideas in sensorimotor experience.

The stark urban-rural disparities (resource access, technology exposure, teacher implementation rates) highlight how pedagogical innovations can exacerbate educational inequality when unequally resourced. This finding echoes broader concerns about the 'digital divide' [27], highlighting the need for equity-conscious implementation strategies that address not only access, but also meaningful participation in digital learning environments. The limited but positive experiences with GeoGebra support Laborde [33] arguments for dynamic geometry software as a bridge between physical manipulation and abstract reasoning. However, the low adoption rate (only 24% of teachers) suggests that technology integration requires more than software availability—it requires pedagogical vision and professional

support.

While this study provides robust evidence through methodological triangulation, several limitations were noted. The study's cross-sectional design and limited experimental component (retention testing) limit causal inferences about constructive proofs improving understanding. Longitudinal or experimental designs would strengthen causal claims. Moreover, this study did not directly measure geometric reasoning ability or proof competency; instead, it relied on proxies (textbook analysis and teacher self-reports). Future research employing validated geometric reasoning assessments would strengthen the empirical foundation. Despite these limitations, the convergence of multiple data sources (triangulation), purposive sample size for a qualitative study and alignment with international literature provide confidence in the study's core findings. In qualitative educational research, explanatory richness and triangulation are accepted indicators of data adequacy [6].

Conclusion

Constructive proofs are still essential to geometry practice and instruction, according to the results. Constructive approaches, in contrast to non-constructive arguments, offer both explanation and verification, allowing students to visualise and actively engage in the reasoning process. Teachers recognised the motivational benefit of constructions and that students demonstrated increased learning and retention while participating in constructive activities. However, it was discovered that obstacles, including time constraints in the curriculum, test anxiety, and issues with practical tools, prevented their consistent use. Therefore, even if constructive proofs are crucial for maintaining the essence of Euclidean geometry, the existing situation does not fully achieve their pedagogical potential. To sum up, constructive proofs are still essential for proving Euclidean theorems. This could change classrooms from places for memorisation to ones that encourage exploration. They are essential to mathematics and mathematical education for their pedagogical advantages and historical continuity. This study recommends that, to ensure balanced learning outcomes, curriculum designers should explicitly call for constructive approaches alongside deductive proofs. Furthermore, without compromising syllabus coverage, professional development should equip teachers with the tools they need to meaningfully incorporate these constructs.

Implications and Suggestions for Further Studies

The major limitation of this study is the exclusion of learners from the role of constructive proofs in Euclidean Geometry propositions. Focussing on textbooks and teachers alone is not enough to reveal the significance of constructive proofs in proving Euclidean Geometry theorems. For future research, researchers can conduct empirical classroom studies that explore the effects of constructive proofs on learners' conceptual understanding, proof competence and geometric reasoning at different educational levels. Such studies will render empirical evidence of how learning outcomes are influenced by constructive approaches in authentic classroom settings. This study has several implications for mathematics education, curriculum development, and research. First, it highlights the need for geometry instruction to move beyond viewing proofs solely as formal deductive arguments by incorporating constructive proofs that enable learners to actively construct,

visualise, and justify geometric relationships. Such an approach can promote deeper conceptual understanding while strengthening learners' reasoning and problem-solving abilities.

References

- [1] P. K. A. Ahiaku, C. Uleanya, G. C. Muyambi, *Rural schools and tech use for sustainability: the challenge of disconnection*, Education and Information Technologies **30** (2025), 12557-12571.
- [2] L. Anwar, M. J. Goedhart, A. Mali, *Learning trajectory of geometry proof construction: Studying the emerging understanding of the structure of Euclidean proof*, Eurasia Journal of Mathematics, Science and Technology Education **19** (2023), em2266.
- [3] K. Boonstra, M. Kool, A. Shvarts, P. Drijvers, *Primary school teachers' practices for fostering embodied abstraction in geometry*, Journal of Mathematics Teacher Education (2025).
- [4] G. A. Bowen, *Document Analysis as a Qualitative Research Method*, Qualitative Research Journal **9** (2009), 27-40.
- [5] J. B. Burgess, *Construction Theorems and Constructive Proofs in Geometry*, Journal for the Philosophy of Mathematics **1** (2024), 23-41.
- [6] J. W. Creswell, J. D. Creswell, *Research Design: Qualitative, Quantitative, and Mixed Methods Approaches*, 5th ed., SAGE Publications, 2017.
- [7] M. De Villiers, *To teach definitions in geometry or teach to define?*, PME conference, 1998.
- [8] N. K. Denzin, & Lincoln, Y, *Handbook of qualitative research* 2nd ed., SAGE Publications, 2000.
- [9] U. Flick, *An Introduction to Qualitative Research* 6th ed., SAGE Publications, 2018.
- [10] G. Hanna, *Proof, Explanation and Exploration: An Overview*, Educational Studies in Mathematics **44** (2000), 5-23.
- [11] G. Hanna, H. N. Jahnke, *Proof and Proving*, in: A.J. Bishop, K. Clements, C. Keitel, J. Kilpatrick, C. Laborde (Eds.) *International Handbook of Mathematics Education: Part 1*, Springer Netherlands, Dordrecht, 1996, 877-908.
- [12] R. Hartshorne, *Geometry: Euclid and beyond*, Springer New York, 2000.
- [13] T. L. Heath, *The thirteen books of Euclid's Elements*, 2nd ed., Dover Publications., 1956.
- [14] J. Hiebert, & Lefevre, P, *Conceptual and procedural knowledge in mathematics: An introductory analysis*. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics*, Lawrence Erlbaum Associates, 1986.
- [15] K. Jones, *Providing a Foundation for Deductive Reasoning: Students' Interpretations when Using Dynamic Geometry Software and Their Evolving Mathematical Explanations*, Educational Studies in Mathematics **44** (2000), 55-85.
- [16] T. Mafugu, *Influences on Mathematics Achievement in Rural Schools: A Nested Study of Underprivileged Learners*, Interchange **56** (2025), 1-15.
- [17] M. T. Malale, H. W. Mbhiza, *Grade 12 rural teachers' Technological Pedagogical Content Knowledge and challenges while using GeoGebra to teach Euclidean geometry*, Frontiers in Education, Frontiers Media SA, 2025, pp. 1703351.

- [18] M. A. Mariotti, *La contribution des environnements technologiques à l'enseignement de la preuve*, Recherches en Didactique des Mathématiques (2024).
- [19] M. A. Mariotti, *Proof and proving in mathematics education*, Handbook of research on the psychology of mathematics education: Past, present and future (2006), 173-204.
- [20] I. Mueller, *Philosophy of mathematics and deductive structure in Euclid's Elements.*, Dover Publications, 2006.
- [21] R. E. Núñez, L. D. Edwards, J. Filipe Matos, *Embodied cognition as grounding for situatedness and context in mathematics education*, Educational Studies in Mathematics **39** (1999), 45-65.
- [22] M. Q. Patton, *Qualitative research & evaluation methods: integrating theory and practice*, 4th ed., SAGE Publications, 2015.
- [23] J. Piaget, *Science of education and the psychology of the child*. Orion Press, (1970).
- [24] D. Sommerhoff, E. Brunner, S. Ufer, "Which Proof for Which Class?" *Factors Influencing the Selection of Mathematical Proofs for Teaching*, Journal für Mathematik-Didaktik **45** (2024), 22.
- [25] S. Sumida, K. Kawata, *An analysis of the learning performance gap between urban and rural areas in sub-Saharan Africa*, South African journal of education **41** (2021), 1-17.
- [26] M. Villiers, *The Role and Function of Proof in Mathematics*, Pythagoras **24** (1990), 17-24.
- [27] M. Warschauer, *Technology and Social Inclusion: Rethinking the Digital Divide*, MIT Press, 2004.
- [28] H.-G. Weigand, K. Hollebrands, M. Maschietto, *Geometry education at secondary level – a systematic literature review*, ZDM – Mathematics Education **57** (2025), 829-843.
- [29] F. Xia, M. J. Nathan, K. E. Schenck, M. I. Swart, *Action Predictions Facilitate Embodied Geometric Reasoning*, Cognitive Science **49** (2025), e70055.
- [30] R. K. Yin, *Case study research and applications: Design and methods* 6th ed., Sage, 2018.
- [31] D. Zhang, Reasoning and proof opportunities in novice teachers' classes: A study of Chinese Eighth-grade geometry lessons, British Educational Research Association Annual Conference 2024 and WERA Focal Meeting, 2024.
- [32] S. C. Kleene, *Introduction to metamathematics*, D. Van Nostrand Company, (1952).
- [33] C. Laborde, Robust and soft constructions: Two sides of the use of dynamic geometry environments, Proceedings of the 10th Asian technology conference in mathematics, Korea National University of Education Cheong-Ju, 2005, pp. 22-35.
- [34] B. Tatira, *An exploration of preservice teachers' mathematics knowledge for teaching in trigonometry at a higher education institution*, University of KwaZulu-Natal, Durban, South Africa, 2020.
- [35] P. Drijvers, H. Kodde-Buitenhuis, M. Doorman, *Assessing mathematical thinking as part of curriculum reform in the Netherlands*, Educational studies in mathematics **102** (2019), 435-456.
- [36] L. Cohen, L. Manion, K. Morrison, *Research methods in education 8th ed.*, routledge London, 2018.