

## MATHEMATICAL MODEL FOR ENZYME: STABILITY ANALYSIS CHAOS DIGNOSIS AND CONTROL

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**Abstract:** Enzyme kinetics mathematically is important since it helps us understand how enzymes work and analyzed interactions with in cells making them essential in medicine, industry and scientific research. We suggest a three-dimensional biomathematical autonomous system, which is made up of three ordinary differential equations, that represent the interaction of the model variables  $N$ ,  $S$ ,  $R$ , where  $N$  is the excited enzyme,  $R$  is the unexcited enzyme and  $S$  is the number of substrate molecules. Real data for sick people was taken from the (Biostudies/Array-Express) website, and the parameters of the suggested system for lipase enzyme kinetics were estimated. The dynamic properties of the suggested system are verified by equilibrium points and it's stability such as roots of characteristic equation, Routh array, Hurwitz stability criteria and Lyapunov function, dissipativity, Hopf bifurcation and Lyapunov dimension based on Lyapunov exponents  $L_1=0.182739$ ,  $L_2=0.229701$  and  $L_3=-2.612439$ . the results of the analysis showed that lipase enzyme kinetics system is unstable and highly chaotic, chaos was efficiently controlled using adaptive control and active control techniques.

### Introduction

Naturally occurring biological catalysts in organisms such as enzymes may accelerate up biochemical reactions all through the metabolism process [6, 13]. Enzymes, which are catalysts or inhibitors in a reaction, are crucial for controlling biological processes, The study of reaction rates, the temporal behaviour of the different reactants, and the factors that affect enzyme kinetics are all necessary to comprehend the function of enzyme kinetics [13]. The books by Murray [14], Rubinow [21] and Michael and Roberts, [26] provide an introduction with a mathematical bent. The analysis attempts to investigate the impact of enzymes on substrates for the creation of the product in order to address the precise theoretical solution for the enzymatic reaction, as a system of nonlinear differential equations, formulate the normalised model of Determine the threshold value to investigate how the substrate is affected by enzymes, complexes, and other factors for the creation of products. [16]. The chaotic dynamics of biological systems can be studied in this way. The behaviour of dynamical systems that are extremely sensitive to the initial conditions and produce widely divergent outcomes is described mathematically as chaos. Biological systems do, in fact, enter chaotic regimes, proving that the idea is not merely theoretical [5]. Nowadays, enzyme kinetics is a

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difficult area of study that integrates contemporary applied mathematics with studies in biotechnology, engineering science, pharmacy, and medicine [19]. In all of these applications, the goal is not to create a biomathematical model that accounts for every understanding of the important interactions within the system. Instead, the objective is to create biomathematical models that distil the core of different interactions so that their results may be better understood. Instead, the objective is to create biomathematical models that distil the core of different interactions so that their results may be better understood. Models get more complex and the mathematics more difficult as more information from biological systems becomes available. In [20], Rössler built a 3-D chaotic system while studying a chemical reaction and discovered it. Many 3-D chaotic systems, including the Arneodo [2], Sprott [22], [3], Liu, et al. [12], Tigan and Opriş [23] and others, have recently been announced in the literature. Other chaotic systems, such as Khattar, et al. [10], Akgul, et al. [1] and Pham [17].

In control theory, the active control method is used when the parameters are available for measurement, while adaptive control is a common control technique used for stabilising systems when the system parameters are unknown [7, 24].

This paper was organized as follows: section2, we described 3-D Enzyme kinetics system. Section3, we analyzed the biomathematical system by its equilibrium, stability analysis, dissipative, bifurcation and Lyapunov exponents. Section 4, numerical and graphical analysis were performed. Section5, we created active and adaptive rules for the chaotic system's control. Section6, the analysis results before and after control are compared. Finally, section7, conclusion for this biomathematical system. Our numerical analysis will enable experimental researchers in modeling the dynamics of biological reacting processes.

## System Description

Biological systems' 3D continuous time nonlinear dynamics was recently constructed by Kadji and colleagues [8]. Considering that  $N$  is the population of enzyme molecules in the excited polar state,  $S$  is the number of substrate molecules, and  $R$  is not. Both the substrate and the enzymes exhibit long-range selective interactions that tend to raise their levels through influx. It is anticipated that the amount of substrate molecules  $S$ , the rate of unexcited enzymes  $R$ , and the activated enzymes' own concentration  $N$  all affect how quickly the activated enzymes rise. Consequently, a system of nonlinear differential equations can be used to characterise the system as shown in formula (1):

$$\left. \begin{aligned} \frac{dN}{dt} &= vNRS - \xi N \\ \frac{dS}{dt} &= \gamma S - vNRS \\ \frac{dR}{dt} &= \xi N - vNRS - \lambda(R - c) \end{aligned} \right\} \dots \dots \dots (1)$$

The values of the parameters are assumed to be:

$$v = 10, \quad \xi = 0.1, \quad \gamma = 5, \quad \lambda = 2 \quad \text{and} \quad c = 0.5 \} \dots \dots \dots (2)$$

The excited enzymes' rate of decay to the ground (or weakly polar) state is denoted by  $\xi$ .

The strength of the nonlinear enzyme-substrate reaction is denoted by  $v$ .

The range at which autocatalytic processes attract the substrate particles is denoted by  $\gamma$ .

The long-range interaction with  $c$  is the source of  $\lambda(R-c)$ , which is the equilibrium concentration of the unexcited enzyme molecules when the excited enzyme and substrate are not present, that is, when  $N=S=0$ .

## Analysis of Systems

The system (1), which is invested in this section, possesses the following properties.

### Equilibrium Point

By setting equations of (1) equal to zero we get two equilibrium points:

$$E_1 = (N, S, R) = (0, 0, c)$$

$$E_2 = (N, S, R) = \left( \frac{\gamma}{vc}, \frac{\xi}{vc}, c \right)$$

### Analysing Stability

#### Roots of characteristic equations

Theorem 1:

- (a) if all the matrix  $J(\vec{u}, \vec{x})$  has eigenvalues with all negative real parts, then  $\vec{x}$  is asymptotically stable.
- (b) if the matrix  $J(\vec{u}, \vec{x})$  possesses at least one eigenvalue with a positive real part, then  $\vec{x}$  is unstable [25].

The Jacobean matrix of system (1) is:

$$J = \begin{bmatrix} vRS - \xi & vNR & vNS \\ -vRS & \gamma - vNR & -vNS \\ \xi - vRS & -vNR & -vNS - \lambda \end{bmatrix}$$

Note that Trace of  $J$  is equal to:

$$vRS - \xi + \gamma - vNR - vNS - \lambda \dots \dots \dots (3)$$

The characteristic equation for biomathematical system (1) is:

$$\begin{aligned} & -K^3 + K^2 (vRS - \xi + \gamma - \lambda - vNS - vNR) \\ & + K (-\gamma vRS + \lambda vRS + \gamma \xi - \xi vNR - \xi \lambda + \gamma vNS + \gamma \lambda - \lambda vNR) \\ & + (-\gamma \lambda vRS + \gamma \xi \lambda - \xi \lambda vNR) = 0 \dots \dots \dots (4) \end{aligned}$$

Given the values of the parameters as in (2), the Jacobean matrix of system (1) at

$$E_1 = (0, 0, 0.5) \text{ is:}$$

$$J_{E_1} = \begin{bmatrix} -0.1 & 0 & 0 \\ 0 & 5 & 0 \\ 0.1 & 0 & -2 \end{bmatrix}$$

And the characteristic equation is:

$$K^3 - 2.9K^2 - 10.3K - 1 = 0 \dots \dots \dots (5)$$

The roots of this characteristic equation (5) are:

$$K_1 = -2, K_2 = -0.1, K_3 = 5$$

Therefore, we deduce that it is unstable system at hyperbolic equilibrium,

$$E_1 = (0, 0, 0.5)$$

Similarly, we find Jacobean matrix and its characteristic equation at  $E_2 = (1, 0.02, 0.5)$ ,

$$J_{E_2} = \begin{bmatrix} 0 & 5 & 0.2 \\ -0.1 & 0 & -0.2 \\ 0 & -5 & -2.2 \end{bmatrix}$$

$$K^3 + 2.2K^2 - 0.5K + 1 = 0 \dots \dots \dots (6)$$

The roots of characteristic equation (6) are:

$$K_1 = -2.54989 \text{ and } K_{2,3} = 0.174944 \mp 0.601306 i$$

We deduced that,  $E_2 = (1, 0.02, 0.5)$  is unstable hyperbolic equilibrium point.

### Routh Stability Criteria

The characteristic equation's roots will have negative real parts if and only if all elements in the first column of the Routh table maintain the same sign. Otherwise, the number of positive real roots will be equal to the number of sign changes in that column [15]. From equation (5) we have:

$$\begin{aligned} a_3 &= 1, & a_2 &= -2.9, & a_1 &= -10.3, & a_0 &= -1 \\ b_1 &= a_1 - \frac{a_0 a_3}{a_2} = -10.64482759 < 0 \\ C_0 &= a_0 = -1 < 0 \end{aligned}$$

**Table 1:** Routh array of system (1)

|       |              |       |
|-------|--------------|-------|
| $K^3$ | 1            | -10.3 |
| $K^2$ | -2.9         | -1    |
| $K^1$ | -10.64482759 | 0     |
| $K^0$ | -1           | 0     |

Since Table (1)'s first column has three negative items, biomathematical system (1) is unstable.

Similarly, we analyze the stability at equilibrium point  $E_2 = (1, 0.02, 0.5)$  and the results are shown in Table 2.

### Hurwitz Stability Criteria

According to this criterion, determinants obtained from the coefficients of the characteristic equation need to be utilized.

A system (1) is considered stable if all the leading principal minors of the square matrix  $J$  are greater than zero. Conversely, if any of these minors are negative or zero, the system (1) is unstable [15].

From equation (5) we find:

$$\begin{aligned}\Delta_1 &= a_2 = -2.9 < 0 \\ \Delta_2 &= \begin{vmatrix} a_2 & a_0 \\ a_3 & a_1 \end{vmatrix} = a_2 a_1 - a_3 a_0 = 30.87 > 0 \\ \Delta_3 &= \begin{vmatrix} a_2 & a_0 & 0 \\ a_3 & a_1 & 0 \\ 0 & a_2 & a_0 \end{vmatrix} = a_2 a_1 a_0 - a_3 a_0^2 = -30.87 < 0\end{aligned}$$

Biomathematical System (1) is unstable since the value of one minor has a value below zero.

Similarly, we analyze the stability at equilibrium point  $E_2 = (1, 0.02, 0.5)$  and the results are shown in Table 2.

### Lyapunov Function

To ensure that the equilibrium is asymptotically stable we have to seek a scalar function of the states and this function is positive definite in a region  $U$  around the equilibrium point:  $V(X) > 0$ , except  $V(X) = 0$ . The existence of a Lyapunov function is sufficient to prove stability (in the sense of Lyapunov) in the region  $U$ .

For system (1) and equation (2), we can utilise the quadratic function, assuming that:

If  $\dot{V}(X)$  is negative definite, the equilibrium is asymptotically stable [18].

For biomathematical system (1) at nonzero equilibrium point and equation (2), we can utilise the quadratic function, assuming that:

$$\begin{aligned}V(N, S, R) &= \frac{1}{2}(N^2 + S^2 + R^2) = 0.6252 > 0 \\ \dot{V}(N, S, R) &= vN^2RS - \xi N^2 + \gamma S^2 - vNRS^2 + \xi NR - vNR^2S - \lambda R^2 + \lambda cR = 0\end{aligned}$$

Since  $\dot{V}(N, S, R) = 0$  therefor, the biomathematical system (1) is unstable.

Similarly, we analyze the stability at equilibrium point  $E_1 = (0, 0, 0.5)$  and the results are shown in Table 2.

**Table 2:** Equilibrium points and it's stability

| Equilibrium points                            | $E_1 = (0, 0, 0.5)$                         | $E_2 = (1, 0.02, 0.5)$                                               |
|-----------------------------------------------|---------------------------------------------|----------------------------------------------------------------------|
|                                               | $K^3 - 2.9K^2 - 10.3K - 1 = 0$              | $K^3 + 2.2K^2 - 0.5K + 1 = 0$                                        |
| Characteristic equation roots and eigenvalues | $K_1 = -2, K_2 = -0.1, K_3 = 5$<br>unstable | $K_1 = -2.54989$ and $K_{2,3} = 0.174944 \pm 0.601306 i$<br>unstable |
| Routh stability criteria                      | $K^3$                                       | $K^3$                                                                |
|                                               | 1                                           | 1                                                                    |
|                                               | $K^2$                                       | $K^2$                                                                |
|                                               | -2.9                                        | 2.2                                                                  |
|                                               | -1                                          | -0.5                                                                 |
| Hurwitz stability criteria                    | $K^1$                                       | $K^1$                                                                |
|                                               | -10.64482759                                | -0.954545454                                                         |
|                                               | $K^0$                                       | $K^0$                                                                |
|                                               | -1                                          | 1                                                                    |
|                                               | 0                                           | 0                                                                    |
| Lyapunov function                             | Unstable                                    | Unstable                                                             |
|                                               | $\Delta_1 = -2.9$                           | $\Delta_1 = 2.2$                                                     |
|                                               | $\Delta_2 = 30.87$                          | $\Delta_2 = -2.1$                                                    |
|                                               | $\Delta_3 = -30.87$                         | $\Delta_3 = -2.1$                                                    |
|                                               | unstable                                    | unstable                                                             |
| Lyapunov function                             | $v(N, S, R) = 0.125 > 0$                    | $v(N, S, R) = 0.6252 > 0$                                            |
|                                               | $\dot{v} = 0$                               | $\dot{v} = 0$                                                        |
|                                               | unstable                                    | unstable                                                             |

### Dissipative

Suppose that [9]:  $f_1 = \frac{dN}{dt}, f_2 = \frac{dS}{dt}, f_3 = \frac{dR}{dt}$

The obtained vector field  $V(\dot{N}, \dot{S}, \dot{R})^T = (f_1, f_2, f_3)^T$

$$\nabla \cdot (\dot{N}, \dot{S}, \dot{R})^T = \frac{\partial f_1}{\partial N} + \frac{\partial f_2}{\partial S} + \frac{\partial f_3}{\partial R} = vRS - \xi + \gamma - vNR - vNS - \lambda$$

Note that from eq. (3) the Trace of the Jacobean matrix  $= \nabla \cdot (\dot{N}, \dot{S}, \dot{R})^T$

The dissipative at the point  $E_1 = (N, S, R) = (0, 0, c)$  is:

$$\frac{\partial f_1}{\partial N} + \frac{\partial f_2}{\partial S} + \frac{\partial f_3}{\partial R} = -\xi + \gamma - \lambda = 2.9 > 0$$

And at the point  $E_2 = (N, S, R) = \left(\frac{\gamma}{vc}, \frac{\xi}{vc}, c\right)$  then

$$\frac{\partial f_1}{\partial N} + \frac{\partial f_2}{\partial S} + \frac{\partial f_3}{\partial R} = -\frac{v\gamma\xi}{(vc)^2} - \lambda = -2.2 < 0$$

Thus, we proved the following proposition (1)

**Proposition 1:** biomathematical system (1) with the estimated parameters as given in equation (2) is not dissipative not conservative at  $E_1 = (0, 0, 0.5)$  and dissipative at  $E_2 = (1, 0.02, 0.5)$

### Bifurcation

A system will exhibit a Hopf bifurcation if it meets the following condition [4]:

The Jacobean matrix contains two purely imaginary eigenvalues and has no other eigenvalues with a real part equal to zero.

$$2 \cdot \left. \frac{d(Re(\mu))}{d\mu} \right|_{\mu = \mu_0} \neq 0$$

From the equation (4) the eigenvalues at the point  $E_2$  are:

$$K_1 = -2.54989, K_{2,3} = 0.174944 \mp 0.601306i$$

The critical points are

$$\lambda_1 = -1.57265, \quad \lambda_2 = 0.105979$$

The calculation results for second condition are shown in table (3):

**Table 3:** The value of second condition of Hopf bifurcation

| K         | $\lambda$ | $\frac{d(\text{Re}(\mu))}{d\mu}$ | $\mu = \mu_0 \neq 0$ |
|-----------|-----------|----------------------------------|----------------------|
| - 2.54989 | -1.57265  | - 0.201976548                    |                      |
|           | 0.105979  | 0.013610957                      |                      |
| 0.174944  | - 1.57265 | - 4.349505852                    |                      |
|           | 0.105979  | 0.29310799                       |                      |

By that we satisfy the two conditions above which means we indicate a Hopf bifurcation. Figure 1. shows the bifurcation diagram of biomathematical system (1), (i) N versus C, (ii) S versus C, (iii) R versus C

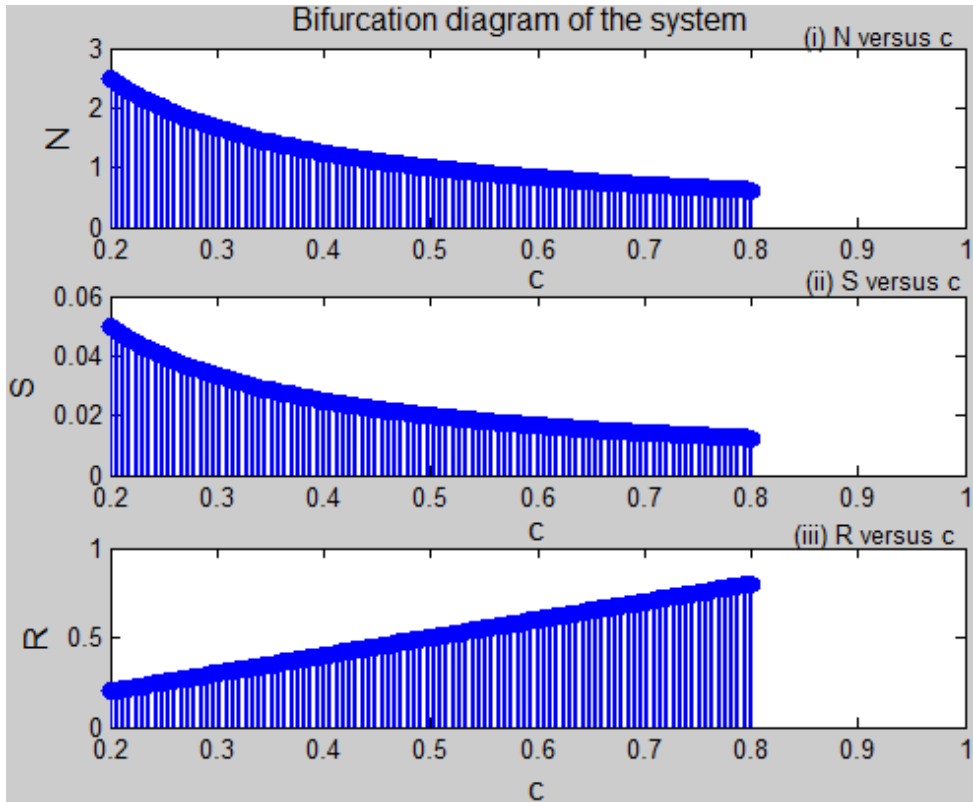


**Figure 1:** bifurcation diagram for system states with parameter, c, at range (0.2,0.8,100) (I) N versus c, (ii) S versus c, (iii) R versus c

### Lyapunov Exponent and Lyapunov Dimension

The dynamical behaviours of a system are determined by Lyapunov exponents, which are the values that demonstrate the exponential attraction or time separation of two adjacent orbits in phase space. When a system has at least one positive Lyapunov exponent, chaos is indicated [11].

Figure 2 illustrates the numerical investigation of the Lyapunov exponents of the biomathematical system (1). Lyapunov exponent settling values at equilibrium point  $E_2$  are as follows:

$$L_1 = 0.182739, L_2 = 0.229701 \quad \text{and} \quad L_3 = -2.612439$$

Furthermore, the Lyapunov dimension is a measure of the degree of a chaotic activity in a biomathematical system (measure the system complexity) it can be calculated as follows:

$$D_L = 2 + \frac{(L_1 + L_2)}{|L_3|} = 2.15788$$

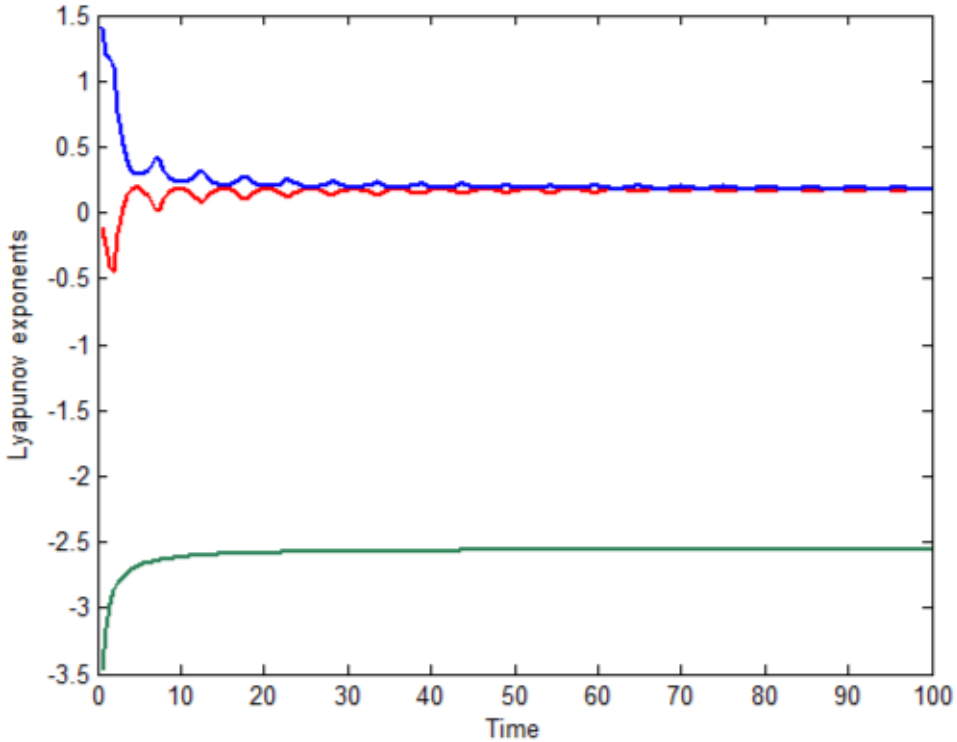


**Figure 2:** dynamics of Lyapunov exponents

### Numerical and graphical analysis

Numerical simulations are used to investigate the hyperchaotic biomathematical system (1) using ord45 Runge-Kutta method with time span 20000. The wave-Form (a)unexcited enzyme R, (b)substrate molecules S, (c)excited enzyme N for the suggested system (1) are shown in Figure 3 which characterized with aperiodic structure. Also, for the phase plane (a):S versus N, (b): R versus N, (c): R versus S and phase portrait (d): R, S, N, Figure 4 illustrates chaotic strange attractors, as the orbit seems to be densely distributed in each graph.

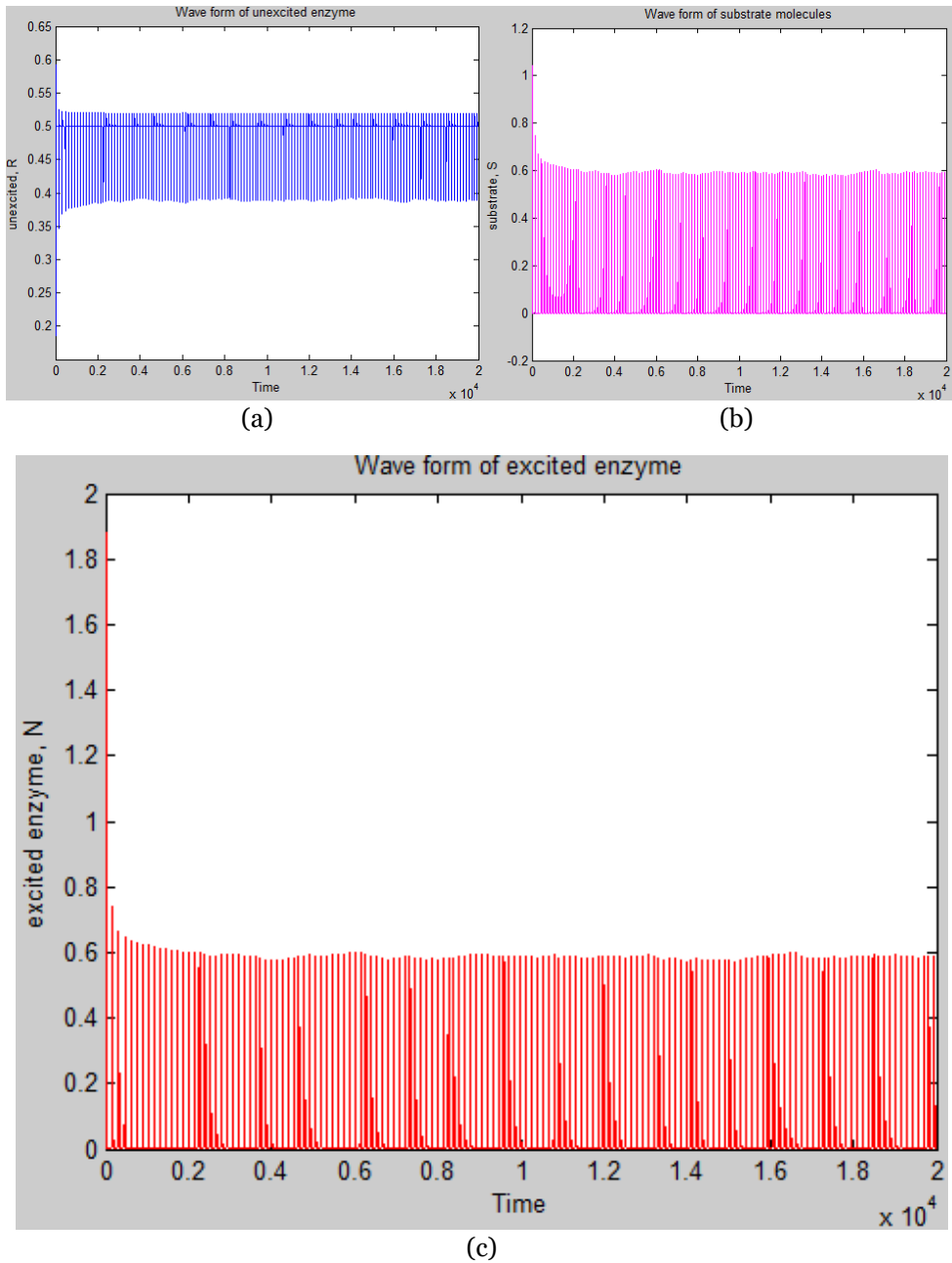


**Figure 3:** Wave Form of the suggested biomathematical autonomous system (1): (a)  $R$  as a function of Time; (b)  $S$  as a function of Time; (c)  $N$  as a function of Time.

Figures 2 and 3 indicate that the suggested biomathematical autonomous system (1) is Chaotic

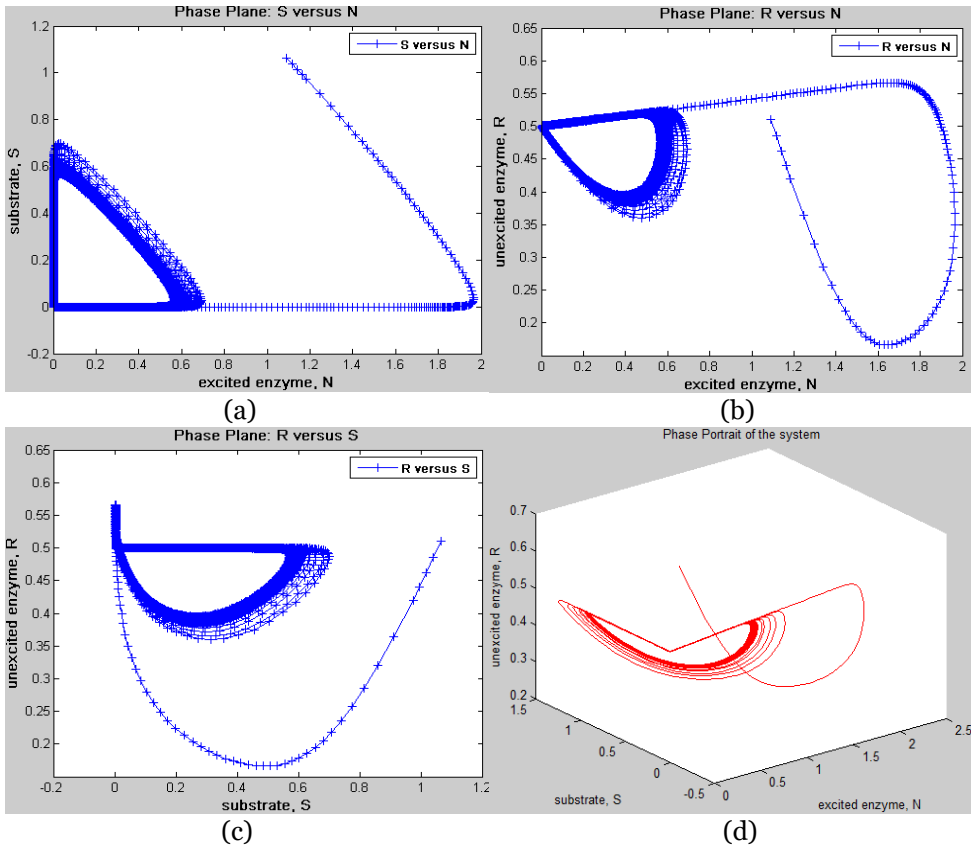


**Figure 4:** phase plane and phase portrait of the suggested biomathematical autonomous system (1): (a) S versus N; (b) R versus N; (c) R versus S; (d) R versus S versus N

## Controlling Technique

### Adaptive Control Technique

#### Theoretical Results

An adaptive control law is created with an unknown parameter  $\gamma$  in order to stabilise a chaotic biomathematical system (1), as follows:

$$\left. \begin{aligned} \dot{N} &= 10NRS - 0.1N + U_1 \\ \dot{S} &= \gamma S - 10NRS + U_2 \\ \dot{R} &= 0.1N - 10NRS - 2(R - 0.5) + U_3 \end{aligned} \right\} \dots \dots \dots (7)$$

When  $U_1, U_2$  and  $U_3$  are feedback controllers. Adaptive control laws are now taken into consideration to ensure that the controlled system (7) converges to the equilibrium point.

$$\left. \begin{aligned} U_1 &= -10NRS + 0.1N - \mu_1 N \\ U_2 &= -\hat{\gamma}S + 10NRS - \mu_2 S \\ U_3 &= -0.1N + 10NRS + 2(R - 0.5) - \mu_3 R \end{aligned} \right\} \dots \dots \dots (8)$$

Where the constants  $\mu_1, \mu_2$  and  $\mu_3$  are positive,  $\hat{\gamma}$  represents an estimation regarding the parameter  $\gamma$ . When the controller (8) is substituted into (7), we obtain:

$$\left. \begin{aligned} \dot{N} &= -\mu_1 N \\ \dot{S} &= (\gamma - \hat{\gamma})S - \mu_2 S \\ \dot{R} &= -\mu_3 R \end{aligned} \right\} \dots \dots \dots (9)$$

Let the parameter estimation error is:

$$\mathbf{E}_\gamma = \gamma - \hat{\gamma} \dots \dots \dots (10)$$

Using (10), system (9) can be written as:

$$\left. \begin{aligned} \dot{N} &= -\mu_1 N \\ \dot{S} &= \mathbf{E}_\gamma S - \mu_2 S \\ \dot{R} &= -\mu_3 R \end{aligned} \right\} \dots \dots \dots (11)$$

The parameter estimate  $\hat{\gamma}$  is modified using the Lyapunov method for determining the update law.

Consider the Lyapunov function as:

$$V(N, S, R) = \frac{1}{2}(N^2 + S^2 + R^2 + \mathbf{E}_\gamma^2) \dots \dots \dots (12)$$

The parameters values are taken as:

$$\mu_1 = 30, \mu_2 = 50, \mu_3 = 30, \mu_4 = 10 \text{ and } \mathbf{E}_\gamma = 0.05 \dots \dots \dots (13)$$

Notice that V is positive definite on  $\mathfrak{R}^4$ , also

$$\dot{\mathbf{E}}_\gamma = -\dot{\hat{\gamma}} \dots \dots \dots (14)$$

Differentiating V (N, S, R), with substituting (11) and (14), we get:

$$\dot{V} = -\mu_1 N^2 - \mu_2 S^2 - \mu_3 R^2 + \mathbf{E}_\gamma (S^2 - \dot{\hat{\gamma}}) \dots \dots \dots (15)$$

In equation (15) we update estimated parameter by:

$$\dot{\hat{\gamma}} = S^2 + \mu_4 \mathbf{E}_\gamma \dots \dots \dots (16)$$

Where  $\mu_4$  is greater than zero. Substituting (16) into (15), we get:

$$\dot{V} = -\mu_1 N^2 - \mu_2 S^2 - \mu_3 R^2 - \mu_4 \mathbf{E}_\gamma^2$$

By using equation (13) at equilibrium points, notice that  $\dot{V}$  is definitely negative on  $\mathfrak{R}^4$ .

Thus, we obtain the following result using eigenvalues, Lyapunov stability, and the Routh and Hurwitz criteria.

**Proposition 2:** by adaptive control (9), where  $\hat{y} = S^2 + \mu_4 \mathbf{E}_y$ . The chaotic system (8) is stabilized at both equilibrium points  $E_1$  and  $E_2$ , and the constants  $\mu_1$ ,  $\mu_2$ ,  $\mu_3$  and  $\mu_4$  are positive.

### Numerical Results

To illustrate the chaotic system under control (7), where the parameters are taken as in equation (13). Figure 5 shows-dynamics of Lyapunov exponents.

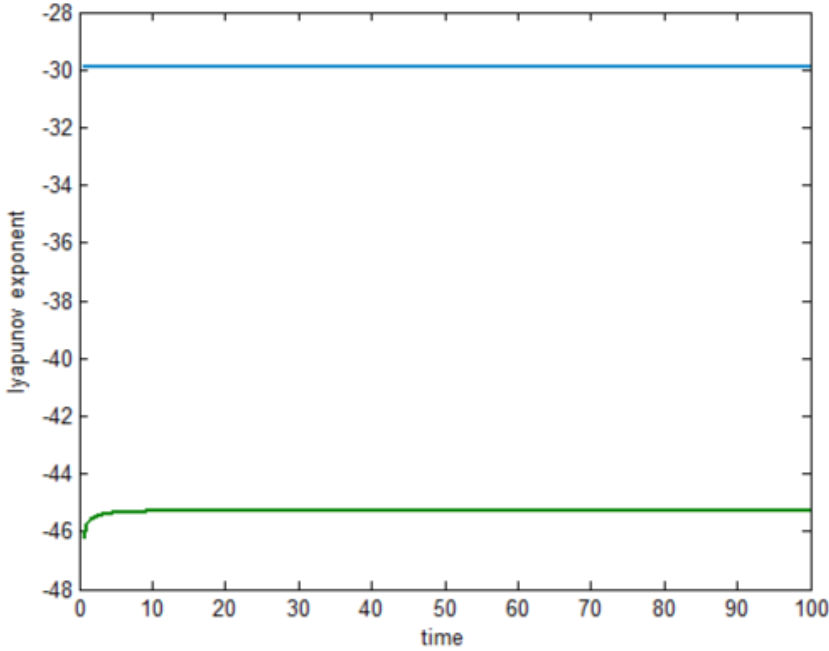


**Figure 5:** dynamics of Lyapunov exponents after adaptive control

### Active Control Technique

#### Theoretical Results

We study the chaos control problem of biomathematical system (1) to regulate it towards equilibrium points,  $E_1 = (N, S, R) = (0, 0, c)$  and  $E_2 = (N, S, R) = \left(\frac{y}{vc}, \frac{\xi}{vc}, c\right)$

We design active control law as follows:

$$\left. \begin{aligned} \dot{N} &= 10NRS - 0.1N + U_1 \\ \dot{S} &= 5S - 10NRS + U_2 \\ \dot{R} &= 0.1N - 10NRS - 2(R - 0.5) + U_3 \end{aligned} \right\} \dots \dots \dots (17)$$

Where  $U_1$ ,  $U_2$  and  $U_3$  are external active control inputs that will be identified later, we consider the control functions as:

$$\left. \begin{aligned} U_1 &= -10NRS + 0.1N - \mu_1 N \\ U_2 &= -5S + 10NRS - \mu_2 S \\ U_3 &= -0.1N + 10NRS + 2(R - 0.5) - \mu_3 R \end{aligned} \right\} \dots \dots \dots (18)$$

Where the constants  $\mu_1$ ,  $\mu_2$  and  $\mu_3$  are positive. Substituting (18) into (17) we get:

$$\left. \begin{aligned} \dot{N} &= -\mu_1 N \\ \dot{S} &= -\mu_2 S \\ \dot{R} &= -\mu_3 R \end{aligned} \right\} \dots \dots \dots (19)$$

Accordingly, we obtain the following result using eigenvalues, Lyapunov stability, Routh array, and Hurwitz stability criteria.

Proposition 3: by active control (18), and  $\mu_1$ ,  $\mu_2$  and  $\mu_3$  are positive constants, The chaotic system (17) is stabilized for both equilibrium points  $E_1$  and  $E_2$ .

### Numerical Results

It is worth mentioning that although the adaptive control is better, the active control was effective and practical in its control of this biomathematical system (1) and the results were satisfactory. To illustrate the controlled chaotic system (18) where the parameters are taken as in equation (13). Dynamics of Lyapunov exponents are displayed in Figure 6.

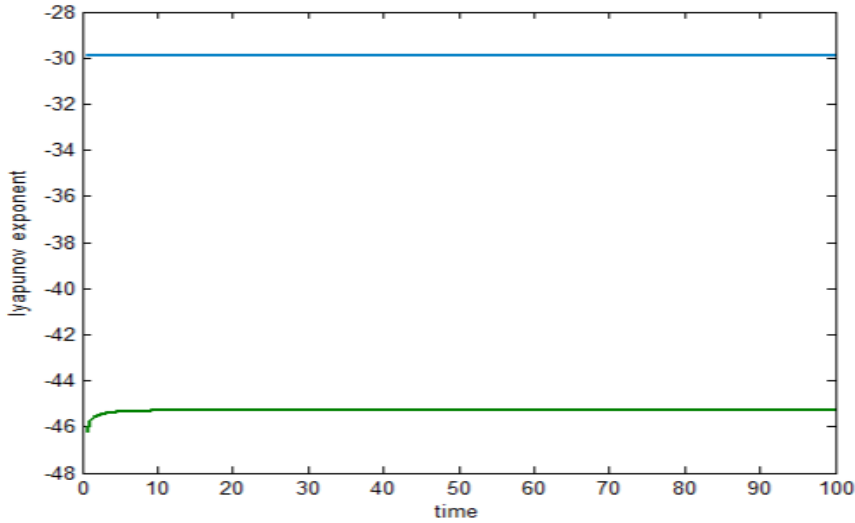


**Figure 6:** dynamics of Lyapunov exponents after active control

Note: the results of stability analysis before and after control are shown at tables (5-8) in section 6.

### Comparison table between two controlling techniques

Since we use two effective techniques for controlling chaos, we will list the complexity, response to change and other properties of both techniques as shown in Table 4.

**Table 4:** Comparison between active and adaptive control

| Aspect             | Active Control                                                | Adaptive Control                                            |
|--------------------|---------------------------------------------------------------|-------------------------------------------------------------|
| OBJECTIVE          | Actively manipulate system to achieve desired output/response | Adjust parameters based on feedback to optimize performance |
| RESPONSE TO CHANGE | Relies on predefined control laws                             | Updates parameters based on system behavior in real time    |
| IMPLEMENTATION     | Requires prior knowledge of system dynamics                   | Designed to handle unknown or time-varying dynamics         |
| COMPLEXITY         | Typically, simpler structure                                  | Can be more complex due to continuous parameter             |
| PERFORMANCE        | Performs well under known/stable conditions                   | Excels in dynamic/uncertain environments                    |

### Comparison of stability before and after controllers

In this section a tables that compares the stability of biomathematical system (1), using different methods before and after adaptive controlling technique and active controlling technique as shown in tables (5, 6, 7, 8).

**Table 5:** A table comparing the stability by Eigenvalues of characteristic equation of system (1) is considered before and after the application of control

| Equilibrium point | Before control             | After adaptive control | After active control |
|-------------------|----------------------------|------------------------|----------------------|
| (0, 0, 0.5)       | $K_1 = -2$                 | $K_1 = -49.95$         | $K_1 = -50$          |
|                   | $K_2 = -0.1$               | $K_2 = -30$            | $K_2 = -30$          |
|                   | $K_3 = 5$                  | $K_3 = -30$            | $K_3 = -30$          |
|                   | unstable                   | stable                 | stable               |
| (1, 0.02, 0.5)    | $K_1 = -2.54989$           | $K_1 = -49.95$         | $K_1 = -50$          |
|                   | $K_2 = 0.174944-0.601306i$ | $K_2 = -30$            | $K_2 = -30$          |
|                   | $K_3 = 0.174944+0.601306i$ | $K_3 = -30$            | $K_3 = -30$          |
|                   | unstable                   | stable                 | stable               |

**Table 6:** A table contrasting system (1)'s stability previously and following control using the Routh array criteria

| Equilibrium point | K              | Before control | K     | After adaptive control | k           | After active control |                |             |       |
|-------------------|----------------|----------------|-------|------------------------|-------------|----------------------|----------------|-------------|-------|
| (0, 0, 0.5)       | K <sup>3</sup> | 1              | -10.3 | K <sup>3</sup>         | 1           | K <sup>3</sup>       | 1              | 3900        |       |
|                   | K <sup>2</sup> | -2.9           | -1    | K <sup>2</sup>         | 109.95      | 3897                 | K <sup>2</sup> | 110         | 45000 |
|                   | K <sup>1</sup> | -10.64482759   | 0     | K <sup>1</sup>         | 3488.132333 | 44955                | K <sup>1</sup> | 3490.909091 | 45000 |
|                   | K <sup>0</sup> | -1             | 0     | K <sup>0</sup>         | 44955       | 0                    | K <sup>0</sup> | 45000       | 0     |
|                   |                | unstable       |       |                        | Stable      |                      |                | stable      |       |
| (1, 0.02, 0.5)    | K <sup>3</sup> | 1              | -0.5  | K <sup>3</sup>         | 1           | 3897                 | K <sup>3</sup> | 1           | 3900  |
|                   | K <sup>2</sup> | 2.2            | 1     | K <sup>2</sup>         | 109.95      | 3897                 | K <sup>2</sup> | 110         | 45000 |
|                   | K <sup>1</sup> | -0.954545454   | 0     | K <sup>1</sup>         | 3488.132333 | 44955                | K <sup>1</sup> | 3490.909091 | 45000 |
|                   | K <sup>0</sup> | 1              | 0     | K <sup>0</sup>         | 44955       | 0                    | K <sup>0</sup> | 45000       | 0     |
|                   |                | unstable       |       |                        | stable      |                      |                | stable      |       |

**Table 7:** System (1) Hurwitz criterion is considered both before and after the application of control

| Equilibrium point | Before control      | After adaptive control             | After active control         |
|-------------------|---------------------|------------------------------------|------------------------------|
| (0, 0, 0.5)       | $\Delta_1 = -2.9$   | $\Delta_1 = 109.95$                | $\Delta_1 = 110$             |
|                   | $\Delta_2 = 30.87$  | $\Delta_2 = 383520.15$             | $\Delta_2 = 384000$          |
|                   | $\Delta_3 = -30.87$ | $\Delta_3 = 1.724114834 * 10^{10}$ | $\Delta_3 = 1.728 * 10^{10}$ |
|                   | Unstable            | stable                             | stable                       |
| (1, 0.02, 0.5)    | $\Delta_1 = 2.2$    | $\Delta_1 = 109.95$                | $\Delta_1 = 110$             |
|                   | $\Delta_2 = -2.1$   | $\Delta_2 = 383520.15$             | $\Delta_2 = 384000$          |
|                   | $\Delta_3 = -2.1$   | $\Delta_3 = 1.724114834 * 10^{10}$ | $\Delta_3 = 1.728 * 10^{10}$ |
|                   | unstable            | stable                             | stable                       |

**Table 8:** System (1)'s Lyapunov function both before and after control

| Equilibrium points | Before control   | After adaptive control  | After active Control   |
|--------------------|------------------|-------------------------|------------------------|
| (0, 0, 0.5)        | $v = 0.125 > 0$  | $v = 0.12625 > 0$       | $v = 0.125 > 0$        |
|                    | $\dot{v} = 0$    | $\dot{v} = -7.525 < 0$  | $\dot{v} = -7.5 < 0$   |
|                    | unstable         | stable                  | stable                 |
| (1, 0.02, 0.5)     | $v = 0.6252 > 0$ | $v = 0.62645 > 0$       | $v = 0.6252 > 0$       |
|                    | $\dot{v} = 0$    | $\dot{v} = -37.545 < 0$ | $\dot{v} = -37.52 < 0$ |
|                    | unstable         | stable                  | stable                 |

## Conclusion

We suggest a three-dimensional biomathematical autonomous system for enzyme kinetics. This system consists of three ordinary differential equations that represent the interaction of the model variables N, S, R, where N is the excited enzyme, R is the unexcited enzyme and S is the number of substrate molecules. Real data was taken from the (Biostudies/Array-Express) website, and the parameters of the mathematical model for lipase enzyme kinetics were estimated. From the data set and the mathematical model, it was found that the kinetics of pancreatic lipase enzyme in the human body is chaotic. The system's behavior was studied in several ways, where we found the equilibrium points of the system and analyzed their stability using the Lyapunov function, Routh array, Hurwitz stability criterion, and the characteristic equation roots. We analyzed chaos using bifurcation and Lyapunov exponents. The system's dissipative was also studied. The results of analysis showed that lipase enzyme kinetics system is unstable according to all used methods and exhibits a Hopf bifurcation, while the "Lyapunov dimension" showed highly chaotic motion with the following Lyapunov exponents:

$$L_1 = 0.182739, L_2 = 0.229701 \text{ and } L_3 = -2.612439$$

After diagnosing the chaotic behavior, we employed adaptive and active control techniques to stabilize the biomathematical autonomous system (1). We get system (1) is stable after implementing adaptive control and active control by using different methods of stability. Adaptive and active controls proved effective in controlling the chaos. So, we conducted a simple comparison between them. All results we get it by using written programs in MATLAB. To obtain the system's behavior before and after control, comparison tables were used. Studying enzyme kinetics is challenging research field incorporating applied mathematics into biotechnology, engineering science and pharmacy and can help researchers predict and improve processes in this fields.

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