

NOVEL 3D-CHAOTIC FINANCIAL MODEL WITH MESH ATTRACTOR: COMPLEXITY ANALYSIS

Zainab Hadi Neamah¹, Ahmed Abdul Mahdi Alawsi², Hashim Marzoog Ali³,
Ahmed M. Ali Ali^{4*}, Ahmed Fahim Al-Baghdadi⁵

Abstract: This article describes a new financial model of chaos that is intended to capture the observed nonlinear behavior in financial markets. The financial systems are highly susceptible to initial conditions, which often exhibit erratic and intricate behavior that is well modeled using chaos theory. By combining complex, nonlinear equations with important financial variables like interest rates, investment returns, and market volatility, this study shows how complex dynamics can emerge in financial markets. The proposed model employs tools from dynamical systems theory, including bifurcation analysis, which is used to assess the stability and probability of market behavior in different scenarios. Simulation results indicate that the critical points at which small alterations to the market lead to significant changes are similar to the real-world patterns of market declines and increases. The findings have important implications for the underlying causes of market instability, and they provide a framework for developing more effective risk management in financial systems.

Introduction

In 1963, Lorenz introduced a three-dimensional model for forecasting weather [4]. This system, recognized as the initial chaotic model, has served as a framework for studies on chaos, which is defined by:

$$\begin{aligned}\dot{x} &= a(y - x) \\ \dot{y} &= cx - y - xz \quad (1) \\ \dot{z} &= xy - bz\end{aligned}$$

In 1966, Rössler proposed dynamical systems [7] that involve different terms and three-dimensional dimensions, which can be expressed as:

$$\begin{aligned}\dot{x} &= -(y - x) \\ \dot{y} &= x + ay \quad (2) \\ \dot{z} &= zx - cz + b\end{aligned}$$

Over the last twenty years, there has been a significant increase in the focus of

¹College of Administration and Economics, University of Kerbala, Kerbala, Iraq.

²Department of Physics, Collage of Science, University of Wasit, Wasit, Iraq.

³College of Administration and Economics, University of Kerbala, Kerbala, Iraq.

⁴ Department of Electronics Techniques, Babylon Technical Institute, Al-Furat Al-Awsat Technical University, Babylon, Iraq. Email: ahmed.ali@atu.edu.iq

⁵ Department of Communications Techniques, Najaf Technical Institute, Al-Furat Al-Awsat Technical University, Iraq.

*Correspondence: ahmed.ali@atu.edu.iq

scientists on solving differential equations related to complex systems, largely driven by the growing use of computers. Researchers have utilized the dynamic behaviors of economic systems to clarify key characteristics of economic data. They have introduced various complex economic models, such as the IS-LM model, Goodwin's model, and the Van Dar pool model, to explore their dynamic attributes. Interest in chaos theory is rising among both specialists and non-specialists, with its relevance extending beyond economics to fields like geometry, physics, ecology, and meteorology. The following areas are under investigation: the precise definition of financial chaos, its similarities to nonlinear dynamics, and methods for identifying financial chaos. Financial economists are developing solution tools based on chaos theory to tackle intricate issues. By employing a specific chaos methodology to assess the potential and constraints within the financial sector, this approach facilitates predictions regarding future changes in financial markets. An intermittent feedback controller, which alternates between periods of stability and chaotic evolution, plays a crucial role in this analysis. This controller aims to assist the financially unstable system in moving from chaos to a state of periodicity within a finely turbulent environment. Various methods, such as the artificial bee colony, grey wolf optimizer, and ant colony optimizer, are utilized to identify the fractional order of financially chaotic systems. The differential equations are transformed into a numerical format and applied to the intricate financial system characterized by the Caputo formula. This numerical analysis has enabled the resolution of certain financial challenges. A numerical simulation was conducted on a financial system represented by differential equations to illustrate the presence of a complex financial system that is compromised due to the intricacies arising from the interconnections of its components. The study describes a financial system featuring new equations for the differential and an Atangana-Baleanu operator [13]. The research was structured to showcase the proposed new system's effectiveness regarding both strengths and weaknesses. Additionally, the trajectories of dynamic financial systems exhibiting chaotic behavior are reconstructed through a data-driven approach that leverages the advantages of the Koopman operator theory and the Takens embedding theory. This method allows for forecasting future events while also managing these systems, including unfamiliar monetary systems. The intricacy of finance is assessed using a novel generalized Caputo derivative, which incorporates three extra elements: interest rates, fluctuations in prices, and investment demand. Different components of the financial system are utilized to identify unique, fixed-parameter solutions. To depict chaotic financial activities, a new set of non-linear ordinary differential equations is used. The points along the null line are analyzed and addressed using the Nystrom model [5]. The analysis of this system involves a concealed secondary attractor and an equilibrium point, utilizing a 2D bifurcation diagram alongside a numerical method to identify a specific point within the system. A bifurcation tree will be presented, showcasing the transition from the first period to chaotic behavior across various parameters. To gain insight into the nonlinear dynamics of this oscillator, the associated unstable periodic solution will be examined. Additionally, several illustrations depicting both stable and unstable periodic motions will be presented [8].

Three Dimensional (3D) Chaotic System Oscillator

Starting with the ideas given in the above introduction citing several authors, one can propose a new model for a chaotic type of financial system, by introducing

different parameters, defining an alternative system with different stability points.

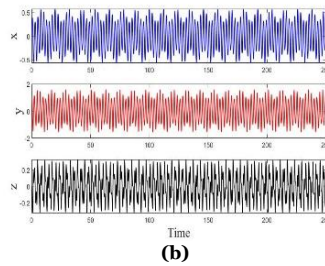
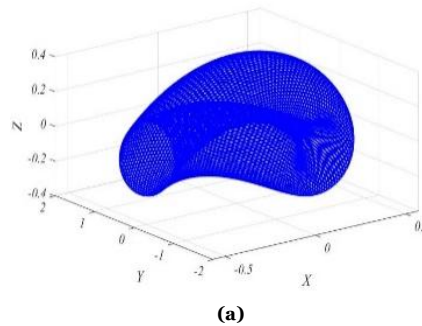
$$\begin{aligned}\dot{x} &= y\dot{y} = R_1(x + yz) \\ \dot{z} &= x^2 + y^2 + R_2(z^2 - R_3x) + R_4\end{aligned}\quad (3)$$

Using comprehensive computer analysis and search methods, we assess the values in Table 1 and investigate the system's parameters.

Table 1: Gives The Values of The Parameters Used in The Numerical Simulation of a Chaotic System.

Parameter	Value
R_1	-3.0
R_2	1.0
R_3	0.1
R_4	- 0.799

Figure 1 Exploration of the three-dimensional phase portrait was provided in the original along with a 2D visualization of the variables and time series of each state variable, Figure 1 (a) 3D Chaotic response, Figure 1 (b) The cut along the y-z plane giving, (c) the projection down the x-axis as seen from above the z-axis angle; and (e) Harmonic amplitudes of x(t), y(t), and z(t). Only five of these figures are extracted and given above. All other figures can be found in the paper [11].



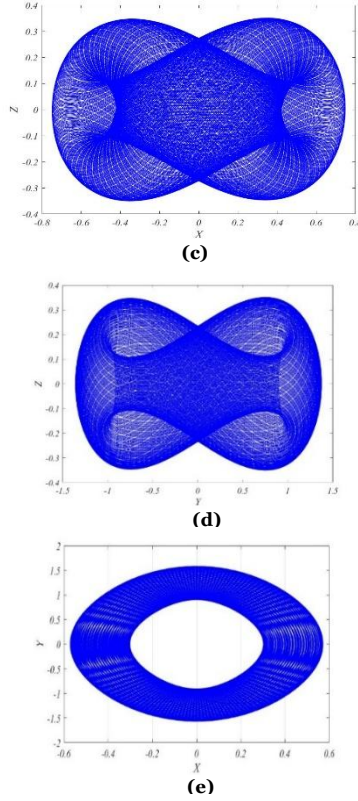


Figure 1: Neighbours and level of freedom L are defined for the discrete approximation of the PDF: $L(a)$ the chaotic regime presented as a projection in three dimensions; (b) the harmonic phase shown in a y - z not-separative plane; (c) presentation of the harmonic phase in a not-separative x - y plane; (d) presentation of harmonic phase in a not-separative x - z plane; and (e) harmonic amplitudes derived from the time series of $x(t)$, $y(t)$, and $z(t)$ [12].

Dynamical Analysis of the System

As was previously mentioned, this system has two equal-pressure points located at $(0.0, \pm\sqrt{R_4})$. Figure 1 shows the behavior of its attractor. The Jacobian matrix is comprised of the following:

$$J = \begin{bmatrix} 0 & 1 & 0 \\ R_1 & R_1 z & R_1 y \\ 2x - R_2 R_3 & 2y & 2z R_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ R_1 & R_1 z & R_1 y \\ 2x - R_2 R_3 & 2y & 2z R_2 \end{bmatrix} (4)$$

Now, one can investigate the stability of equilibrium points by testing its eigenvalues of this system by enhancing the characteristic equation. One can find the Lyapunov exponents for the point $(0.0, \sqrt{R_4})$ are: $LE_1 = 1.7877 + 0.0000i$, $LE_2 = -1.3408 + 1.0965i$ and $LE_3 = -1.3408 - 1.0965i$, respectively. Similarly, for the second equilibrium point $(0.0, -\sqrt{R_4})$ the Lyapunov exponents are: $LE_1 = -1.7877 + 0.0000i$, $LE_2 = 1.3408 + 1.0965i$ and $LE_3 = 1.3408 - 1.0965i$. By comparing the

results, one can find that the equilibrium point $(0.0, \sqrt{R_4})$ is stable while the other one $(0.0, -\sqrt{R_4})$ is unstable [18].

Now, by choosing the same control parameters in Table 1, the Kaplan-Yorke dimension expressed as:

$$D_L = k + \frac{1}{|\lambda_{k+1}|} \sum_{j=1}^k \lambda_j \quad (5)$$

where k is the largest integer satisfying the following conditions $\sum_{j=1}^k \lambda_j \geq 0$ and $\sum_{j=1}^{k+1} \lambda_j \leq 0$. The Kaplan-Yorke dimension reflects the complexity of the attractor, serving as a metric for the level of disorder among the points located on the attractor [3]. Thus, this is derived from the Lyapunov spectrum, we find that $\sum_{j=1}^{k+1} \lambda_j = -3 < 0$, which emphasizes that system (3) is dissipative. The computed fractional dimension with such system parameters setting is:

$$D_L = 3 + \frac{\lambda_1 + \lambda_2 + \lambda_3}{|\lambda_4|} = 4.0321 \quad (6)$$

Obviously, equation (6) indicates that the dissipative system under investigation displays chaotic behavior [15].

Transition to Chaos

To check the sensitivity of the motion of this chaotic behavior, we choose some set of control parameters. Here, we investigate from $(A = 0.25, B = -0.05, C = -0.11)$, and a sequence of period-doubling bifurcations to chaos as shown in Figure 2 will result from the reduction of R_4 below (-0.1) . We subject the time series resulting from R_4 decreasingly from $(-3 < R_4 < -0.1)$ and increasingly again at $(-27 < R_4 < -0.6)$ to inspection for chaos. When R_4 is decreased, we observe a transition to a period-1 limit cycle by the Hopf phenomenon, followed by a transformation to chaos through period-doubling bifurcations step by step as shown in Figure 2. This demonstrates that the system is capable of fixed-point motion on an invariant cycle, with possible transitions to period-doubling and even on to chaotic motion by appropriate variations in the control parameter [1, 2, 14, 16, 17].

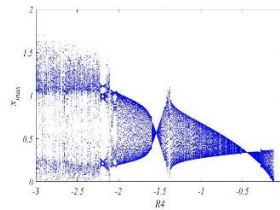


Figure 2: (a) Bifurcation Diagram Depicted y-Axis with the R_4 Control Parameter.

For Y_{Max} the control parameter that introduces a complex behavior into the range $(-3 < 0.1)$ has the same behavior as the R_4 control parameter above. Figure 3 illustrates a schematic of a bifurcation tree.

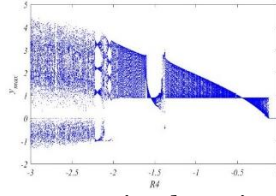


Figure 2: (b) Bifurcation Diagram Depicted y-Axis with the R4 Control Parameter.

The 3-Bifurcation Plot Illustrated the Y-Axis with the R_4 Paramater Controlling It.

The behavior of this chaotic system is observed to differ based on the specific parameters altered. For example, changing the values of R_3 and R_11 will lead to a stable state, as illustrated in Figure 4 (a). A small increase in the parameters will result in the system exhibiting a limit cycle, similar to the example in Figure 4 (b). Additionally, when the parameters are below their default values, the system exhibits a tour-like behavior, demonstrated in Figure 4 (c). In contrast, Figure 4 (d) shows the inconsistent behavior of the system.avior of the system with its default parameters values [20].

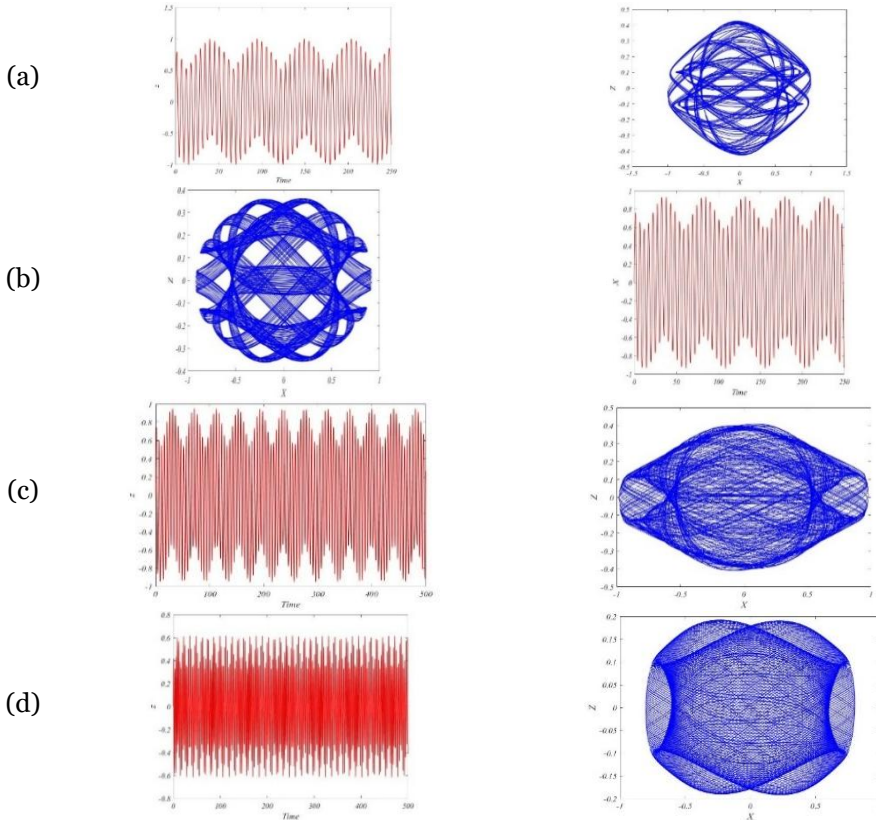


Figure 3: Dynamical systems: (a) The behavior at the first and fourth points is 0.8. (b) The behavior of the system at $R_3 = -2.99$ and $R_3 = 1.0$. (c) The behavior of the system at $R_1 = 2.99$, $R_4 = 0.8$. (d) Chaotic system at $1.5 \leq R_1$ and $0.799 \leq R_4$ [6, 9, 10, 19].

Hidden Attractor

In this system, when fixed equilibrium is available, we find the basin of attraction by employing similar parameter values given in Table 1. Figures 5 demonstrate the basin of attraction for the system. In chaotic systems, initial conditions are of great importance, which points to the possibility of attractors in complex systems.

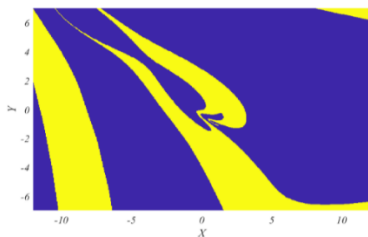


Figure 4: The basin of attraction's sections for a specific control parameter are listed in Table 1 $x_0(0) = 0.0, y_0 = 0.0, z_0(0) = 1.0$. From this, it demonstrates the chaotic yellow and unbound blue regions.

Conclusions

The model's ability to produce bistable and erratic attractors highlights the traditional linear models' limitations in predicting financial results. It demonstrates that financial systems are inherently dependent on initial conditions, which makes it difficult to make long-term predictions with confidence. However, by recognizing critical areas and comprehending the system's random nature, the model provides significant insight into the regulation of markets and risk management strategies.

References

- [1] L. Ding, Q. Ding, *A Novel Image Encryption Scheme Based on 2D Fractional Chaotic Map, DWT and 4D Hyper-chaos*, Electronics **9** (2020), 1280.
- [2] U. Erkan, A. Toktas, Q. Lai, *2D hyperchaotic system based on Schaffer function for image encryption*, Expert Systems with Applications **213** (2023), 119076.
- [3] P. Frederickson, J. L. Kaplan, E. D. Yorke, J. A. Yorke, *The Liapunov Dimension of Strange Attractors*, Journal of Differential Equations **49** (1983), 185-207.
- [4] Y. Hu, *3D Model Encryption Algorithm by Parallel Bidirectional Diffusion and 1D Map with Sin and Logistic Coupling*, Comput. Syst. Sci. Eng. **47** (2023), 1819-1838.
- [5] Z.-W. Huang, N.-R. Zhou, *Image encryption scheme based on discrete cosine Stockwell transform and DNA-level modulus diffusion*, Optics & Laser Technology **149** (2022), 107879.
- [6] K. Jain, A. Aji, P. Krishnan, *Medical Image Encryption Scheme Using Multiple Chaotic Maps*, Pattern Recognit. Lett. **152** (2021), 356-364.
- [7] J. Li, W. Zhang, B. Zhao, *3D medical model encryption based on five-dimensional hyperchaotic systems with 3D Arnold transform and selectable multiple spiral arrangements*, J. Supercomput. **81** (2024), 39.

- [8] X. Li, J. Mou, S. Banerjee, Z. Wang, Y. Cao, *Design and DSP implementation of a fractional-order detuned laser hyperchaotic circuit with applications in image encryption*, Chaos Solitons Fractals **159** (2022), 112133.
- [9] H. Lin, C. Wang, J. Sun, X. Zhang, Y. Sun, H. H. C. Iu, *Memristor-coupled asymmetric neural networks: Bionic modeling, chaotic dynamics analysis and encryption application*, Chaos Solitons Fractals **166** (2023), 112905.
- [10] A. Manzoor, A. H. Zahid, M. T. Hassan, *A New Dynamic Substitution Box for Data Security Using an Innovative Chaotic Map*, IEEE Access **10** (2022), 74164-74174.
- [11] Y. Pourasad, R. Ranjbarzadeh, A. Mardani, *A New Algorithm for Digital Image Encryption Based on Chaos Theory*, Entropy **23** (2021), 341.
- [12] W. Song, C. Fu, M. Tie, C.-W. Sham, J. Liu, H.-f. Ma, *A fast parallel batch image encryption algorithm using intrinsic properties of chaos*, Signal Processing: Image Communication **102** (2022), 116628.
- [13] C. Sparrow, *The Lorenz Equations: Bifurcations, Chaos, and Strange Attractors*, New York, NY: Springer New York, 1982.
- [14] X. Wang, S. Gao, *Application of matrix semi-tensor product in chaotic image encryption*, J. Franklin Inst. **356** (2019), 11638-11667.
- [15] Z. Wu, P. Pan, C. Sun, B. Zhao, *Plaintext-Related Dynamic Key Chaotic Image Encryption Algorithm*, Entropy **23** (2021), 1159.
- [16] H. Xiang, L. Liu, *A random irregular blocking image encryption algorithm based on improved digital chaotic maps at bit level*, Int. J. Bifurcation Chaos Appl. Sci. Eng. **32** (2022), 2250054.
- [17] W. Xingyuan, G. Suo, Y. Xiaolin, Z. Shuang, W. Mingxu, *A new image encryption algorithm with cantor diagonal scrambling based on the PUMCML system*, Int. J. Bifurcation Chaos Appl. Sci. Eng. **31** (2021), 2150003.
- [18] A. H. Zahid, M. J. Arshad, M. Ahmad, N. F. Soliman, W. El-Shafai, *Dynamic S-Box Generation Using Novel Chaotic Map with Nonlinearity Tweaking*, CMC, Comput. Mater. Continua **75** (2023), 3011-3026.
- [19] X. Zhang, L. Zhang, *Multiple-image encryption algorithm based on chaos and gene fusion*, Multimedia Tools Appl. **81** (2022), 20021-20042.
- [20] Y. Zhang, Y. He, J. Zhang, X. Liu, *Multiple Digital Image Encryption Algorithm Based on Chaos Algorithm*, Mobile Networks and Applications **27** (2022), 1349-1358.