

SOME CONCEPTS BY NANO GRILL GENERALIZED DELTA OPEN SET IN TOPOLOGICAL SPACE

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Abstract: The aim of this paper is to introduce a new class of sets called Nano grill generalized delta open sets in topological spaces and some of their basic properties are investigated a new class of sets lies between $N-g\delta$ - (Reg, α , pre, semi, β)-open set. We obtain their characterizations, their basic properties and their relationships with other types of sets in topological spaces.

Introduction

In 1947, the concept of grill set introduced by Choquet [2]. In 2014, the concept of Nano generalized open (closed) sets introduced by Bhuvaneswari and Gnanapriya [1] and introduce Nano generalized pre closed set. Plays a significant role in general topology and may research papers were published which deal with different types of Nano generalized open (closed) sets. Also Thanga in 2014 introduced the concept of $N-g-\alpha$ -closed($N-g-\alpha$ -open)set [3]. These concepts motivated us to define a new class of set, called the Nano generalized delta open sets. In 2015, A prior study established the notion of $N-\delta$ -open sets. In 2022, Saliem and Shihab [5] introduced the concept of $N-\delta$ -open set is ($N-\delta$ -pre and $N-\delta$ -semi) open set and $N-g-\delta-\alpha$ -open (resp. $N-g-\delta$ -pre-open, $N-g-\delta$ -semi-open, $N-g-\delta-\beta$ -open and $N-g-\delta$ -Reg-open) sets. Throughout this paper, we find the concept of Nano Grill open set and explain the relation between some sets such that Nano-G- δ (α , pre, semi, β) open set with them complements. Also, we introduce the concept of Nano Grill Generalized closed set and explain the relation between same sets. In addition, we offer the notion of Nano grill Generalized δ , Nano grill Generalized δ closed set with some sets such as $N-G-g-\delta$ -Reg-closed (resp. $N-G-g-\delta-\alpha$ -closed, $N-G-g-\delta$ -pre-closed, $N-G-g-\delta$ -semi-closed and $N-G-g-\delta-\beta$ -closed) sets.

Preliminaries

Let us recall the following definitions which are useful in the:

Definition 2.1: [2]

A collection G of non-empty subsets of a topological space X is named a Grill if it is satisfying the following:

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i. If $A \in G$ and $A \subseteq \mathcal{B} \subseteq X$, then $\mathcal{B} \in G$.

ii. $A, \mathcal{B} \in X$ and $A \cup \mathcal{B} \in G$, then $A \in G$ or $\mathcal{B} \in G$.

Let X be a nonempty set, then the following are Grill on Z . [6]

i. $\emptyset$ And $P(X) \setminus \{\emptyset\}$ are trivial examples of Grills on X .

ii. G_∞ the Grill of all infinite subsets of Z (where X is infinite set).

iii. G_{co} the Grill of all uncountable subsets of X .

iv. $G_p = \{A: A \in P(X), p \in A\}$ is a specific point grill on X .

v. $G_A = \{\mathcal{B}: \mathcal{B} \in P(X), A \cap \mathcal{B} \neq \emptyset\}$.

If $(X, \mathcal{T})$ is a topological space, then the family of all non-nowhere dense subsets called $G = \{A: \text{int}(\text{cl}(A)) \neq \emptyset\}$ is one of kinds of grill on X .

Suppose that G is a grill on $(X, \mathcal{T})$. A mapping $\phi: P(\mathcal{T})$ is referred to as $\phi(A) = \{x \in X : A \cap u \in G \text{ for every } u \in \mathcal{T}_{(x)}\}$ for every $A \in P(X)$ and $\mathcal{T}_{(x)}$ which is all open set containing x .

A mapping $F: P(X) \rightarrow P(X)$ is referred to as $F(A) = A \cup F(A)$ for every $A \in P(X)$. [4]

The map F kuratowski closure axioms are satisfied:

i. $F(\emptyset) = \emptyset$.

ii. When $A \subseteq \mathcal{B}$, then $F(F(A)) = F(A)$.

iii. When $A, \mathcal{B} \subseteq X$, then $F(A \cup \mathcal{B}) = F(A) \cup F(\mathcal{B})$.

Definition 2.2: [7]

Let U be the universe, R be an equivalence relation on U and $\mathcal{T}_R(x) = \{U, \emptyset, B_R(x), L_R(x), U_R(x)\}$ satisfies the following axioms:

1. $\emptyset$ and $U \in \mathcal{T}_R(x)$

2. The union of objects of any sub collection of $\mathcal{T}_R(x)$ is in $\mathcal{T}_R(x)$

3. The intersection of the objects of any sub collection of $\mathcal{T}_R(x)$ is in $\mathcal{T}_R(x)$.

Thus $\mathcal{T}_R(x)$ is topology on U be called Nano-topology on U with respect to X . we call $(U, \mathcal{T}_R(x))$ as the Nano-Topological space. The members of $\mathcal{T}_R(x)$ is said to be Nano-open sets.

Definitions 2.3:

Let $(U, \mathcal{T}_R(x))$ be a Nano topological space and $F \subseteq U$, then F is said to be:

1. Nano δ -open set if $F = N \text{int}_\delta(F)$

2. Nano Reg open set if $F = N \text{int} (N \text{cl}(F))$.

3. Nano- α -open set if $F \subseteq N \text{int} (N \text{cl}(N \text{int}(F)))$.

4. Nano-pre-open set if $F \subseteq N \text{int} (N \text{cl}(F))$.

5. Nano-semi-open set if $F \subseteq N \text{cl} (N \text{int}(F))$.

6. Nano- β -open set if $F \subseteq N \text{cl} (N \text{int}(N \text{cl}(F)))$.

Definition 2.4:

Let $(U, \mathcal{T}_R(x))$ be a Nano topological space and $F \subseteq U$, then the $N\delta$ -interior (resp. $N\delta$ -closure) of F is denoted by $N \text{int}_\delta(F) = \cup \{D: D \text{ is } N \text{Reg-open set and } D \subseteq F\}$ (resp. $N \text{cl}_\delta(F) = \cup \{X \in U: N \text{int}(N \text{cl}(D) \cap F \neq \emptyset \text{ and } X \in D\}$).

Definitions 2.5:

Let $(U, \mathcal{T}_R(x))$ be a Nano topological space and $F \subseteq U$, then F is said to be:

1. N - δ -Reg-open if $F = N_{int\delta}(Ncl_{\delta}(F))$.
2. N - δ - α -open if $F \subseteq N_{int\delta}(Ncl_{\delta}(N_{int\delta}(F)))$.
3. N - δ -pre-open if $F \subseteq N_{int\delta}(Ncl_{\delta}(F))$.
4. N - δ -semi-open if $F \subseteq Ncl_{\delta}(N_{int\delta}(F))$.
5. N - δ - β -open if $F \subseteq Ncl_{\delta}(N_{int\delta}(Ncl_{\delta}(F)))$.

Definitions 2.6

Let $(U, \mathcal{T}_R(X))$ be a Nano topological space, a subset F of $(U, \mathcal{T}_R(X))$ is said to be:

1. Nano Generalized closed set (N -g-closed) if $Ncl(F) \subseteq D$ where $F \subseteq D$ and D is N -open set in $(U, \mathcal{T}_R(X))$. [1]
2. N -g- δ -closed set if $Ncl_{\delta}(F) \subseteq D$ where $F \subseteq D$, and D is N -open in $(U, \mathcal{T}_R(X))$. [5]
3. N -g-Reg-closed set if N -Reg- $cl(F) \subseteq D$ where $F \subseteq D$, and D is N -open in $(U, \mathcal{T}_R(X))$.
4. N -g- α -closed set if N - α - $cl(F) \subseteq D$ where $F \subseteq D$, and D is N -open in $(U, \mathcal{T}_R(X))$.
5. N -g-semi-closed set if N -semi- $cl(F) \subseteq D$ where $F \subseteq D$, and D is N -open in $(U, \mathcal{T}_R(X))$.
6. N -g-pre-closed set if N -pre- $cl(F) \subseteq D$ where $F \subseteq D$, and D is N -open in $(U, \mathcal{T}_R(X))$.
7. N -g- β -closed set if N - β - $cl(F) \subseteq D$ where $F \subseteq D$, and D is N -open in $(U, \mathcal{T}_R(X))$.

Definition 2.7:

Let $(U, \mathcal{T}_R(X))$ be a Nano topological space and $V \subseteq U$ then V is said to be N -g-open set (resp. N -g- δ -open, N -g-Reg-open, N -g- α -open, N -g-pre-open, N -g-semi-open and N -g- β -open) sets if its complement N -g-closed set (resp. N -g- δ -closed, N -g-Reg-closed, N -g- α -closed, N -g-pre-closed, N -g-semi-closed and N -g- β -closed) sets.

Definitions 2.8: [5]

Let $(U, \mathcal{T}_R(X))$ be a Nano topological space and $F \subseteq U$, then F is said to be:

1. N -g- δ - α -closed set if N - δ - α - $cl(F) \subseteq D$ where $F \subseteq D$, and D is N -open in $(U, \mathcal{T}_R(X))$.
2. N -g- δ -pre-closed set if N - δ -pre- $cl(F) \subseteq D$ where $F \subseteq D$, and D is N -open in $(U, \mathcal{T}_R(X))$.
3. N -g- δ -semi-closed set if N - δ -semi- $cl(F) \subseteq D$ where $F \subseteq D$, and D is N -open in $(U, \mathcal{T}_R(X))$.
4. N -g- δ - β -closed set if N - δ - β - $cl(F) \subseteq D$ where $F \subseteq D$, and D is N -open in $(U, \mathcal{T}_R(X))$.
5. N -g- δ -Reg-closed set if N - δ -Reg- $cl(F) \subseteq D$ where $F \subseteq D$, and D is N -open in $(U, \mathcal{T}_R(X))$.

Definition 2.9: [5]

Let $(U, \mathcal{T}_R(X))$ be a Nano topological space and $V \subseteq U$ then V is said to be N -g- δ - α -open (resp. N -g- δ -pre-open, N -g- δ -semi-open, N -g- δ - β -open and N -g- δ -Reg-open) sets if its complement N -g- δ - α -closed (resp. N -g- δ -pre-closed, N -g- δ -semi-closed, N -g- δ - β -closed and N -g- δ -Reg-closed) sets.

Example 2.10:

Let $U = \{1, 2, 3, 4\}$ with $U/R = \{\{1\}, \{2, 4\}\}$ and let $X = \{1, 2\}$ then $\mathcal{T}_R(X) = \{U, \emptyset, \{1\},$

$\{2, 4\} \{1, 2, 4\}$ and $\mathcal{T}_R(X)^c = \{U_1, \emptyset, \{2, 3, 4\}, \{1, 3\}, \{3\}\}$ then:

Table 2-1: Explain the Relation Between N-G- δ -Closed Sets.

F	N-g-closed	N-g- δ -closed	N-g- δ -Reg closed	N-g- δ - α -closed	N-g- δ -pre closed	N-g- δ -semi closed	N-g- δ - β -closed
$\{1\}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\{1\}$	$\{1\}$
$\{2\}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\{2\}$	$\{2\}$	$\{2\}$
$\{3\}$	$\{3\}$	$\{3\}$	$\{3\}$	$\{3\}$	$\{3\}$	$\{3\}$	$\{3\}$
$\{4\}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\{4\}$	$\{4\}$	$\{4\}$
$\{1, 2\}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$\{1, 3\}$	$\{1, 3\}$	$\{1, 3\}$	$\{1, 3\}$	$\{1, 3\}$	$\{1, 3\}$	$\{1, 3\}$	$\{1, 3\}$
$\{1, 4\}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$\{2, 3\}$	$\{2, 3\}$	$\{2, 3\}$	$\{2, 3\}$	$\{2, 3\}$	$\{2, 3\}$	$\{2, 3\}$	$\{2, 3\}$
$\{2, 4\}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\{2, 4\}$	$\{2, 4\}$
$\{3, 4\}$	$\{3, 4\}$	$\{3, 4\}$	$\{3, 4\}$	$\{3, 4\}$	$\{3, 4\}$	$\{3, 4\}$	$\{3, 4\}$
$\{1, 2, 3\}$	$\{1, 2, 3\}$	$\{1, 2, 3\}$	$\{1, 2, 3\}$	$\{1, 2, 3\}$	$\{1, 2, 3\}$	$\{1, 2, 3\}$	$\{1, 2, 3\}$
$\{1, 2, 4\}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$\{1, 3, 4\}$	$\{1, 3, 4\}$	$\{1, 3, 4\}$	$\{1, 3, 4\}$	$\{1, 3, 4\}$	$\{1, 3, 4\}$	$\{1, 3, 4\}$	$\{1, 3, 4\}$
$\{2, 3, 4\}$	$\{2, 3, 4\}$	$\{2, 3, 4\}$	$\{2, 3, 4\}$	$\{2, 3, 4\}$	$\{2, 3, 4\}$	$\{2, 3, 4\}$	$\{2, 3, 4\}$
X	X	X	X	X	X	X	X
$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

Example 2.11: from example 2.10, we get

Table 2-2: Explain the Relation Between N-G- δ -Open Sets.

F	N-g-open	N-g- δ -open	N-g- δ -Reg open	N-g- δ - α -open	N-g- δ -pre open	N-g- δ -semi open	N-g- δ - β -open
$\{1\}$	$\{1\}$	$\{1\}$	$\{1\}$	$\{1\}$	$\{1\}$	$\{1\}$	$\{1\}$
$\{2\}$	$\{2\}$	$\{2\}$	$\{2\}$	$\{2\}$	$\{2\}$	$\{2\}$	$\{2\}$
$\{3\}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$\{4\}$	$\{4\}$	$\{4\}$	$\{4\}$	$\{4\}$	$\{4\}$	$\{4\}$	$\{4\}$
$\{1, 2\}$	$\{1, 2\}$	$\{1, 2\}$	$\{1, 2\}$	$\{1, 2\}$	$\{1, 2\}$	$\{1, 2\}$	$\{1, 2\}$
$\{1, 3\}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\{1, 3\}$	$\{1, 3\}$
$\{1, 4\}$	$\{1, 4\}$	$\{1, 4\}$	$\{1, 4\}$	$\{1, 4\}$	$\{1, 4\}$	$\{1, 4\}$	$\{1, 4\}$
$\{2, 3\}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$\{2, 4\}$	$\{2, 4\}$	$\{2, 4\}$	$\{2, 4\}$	$\{2, 4\}$	$\{2, 4\}$	$\{2, 4\}$	$\{2, 4\}$
$\{3, 4\}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$\{1, 2, 3\}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\{1, 2, 3\}$	$\{1, 2, 3\}$	$\{1, 2, 3\}$
$\{1, 2, 4\}$	$\{1, 2, 4\}$	$\{1, 2, 4\}$	$\{1, 2, 4\}$	$\{1, 2, 4\}$	$\{1, 2, 4\}$	$\{1, 2, 4\}$	$\{1, 2, 4\}$
$\{1, 3, 4\}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\{1, 3, 4\}$	$\{1, 3, 4\}$	$\{1, 3, 4\}$
$\{2, 3, 4\}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\{2, 3, 4\}$	$\{2, 3, 4\}$
X	X	X	X	X	X	X	X
$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

On Nano Grill Generalized Delta Open Set

Definition 3.1

Let $(U, \mathcal{T}_{R(X)G})$ be a Nano-Grill topological space and $F \subseteq U$. then, F is said to be:

1. Nano-G- α open if $F \subseteq \text{Nint}_G(\text{Ncl}_G(\text{Nint}(F)))$.
2. Nano-G-preopen if $F \subseteq \text{Nint}_G(\text{Ncl}_G(F))$.
3. Nano-G-semi open if $F \subseteq \text{Ncl}_G(\text{Nint}_G(F))$.
4. Nano-G- β open if $F \subseteq \text{Ncl}_G(\text{Nint}_G(\text{Ncl}_G(F)))$.
5. Nano-G-Reg open if $F = \text{Nint}_G(\text{Ncl}_G(F))$.
6. Nano-G- δ open if $F = \text{Nint}_G(\delta_G(F))$.

We can explain the relation between some types of Nano-G-open sets by the diagram.

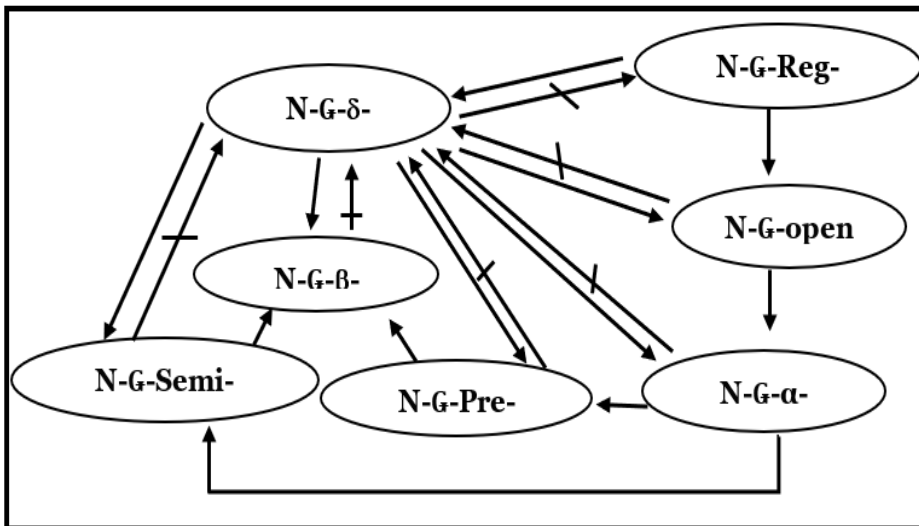


Diagram 3.1: Explain the Relation Between Some Type of N-G-Open Sets.

Definition 3.2

Let $(U, \mathcal{T}_{R(X)G})$ be a Nano Grill topological space and $F \subseteq U$, then F is called Nano-G-Reg closed (Nano-G- α -closed, Nano-G- β -closed, Nano-G-pre-closed, Nano-G-semi-closed, Nano-G- δ -closed) sets, if its complement is Nano-G-Reg-open (Nano-G- α -open, Nano-G- β -open, Nano-G-pre-open, Nano-G-semi-open, Nano-G- δ -open) sets. We can explain the relation between some types of Nano Grill closed sets by the diagram.

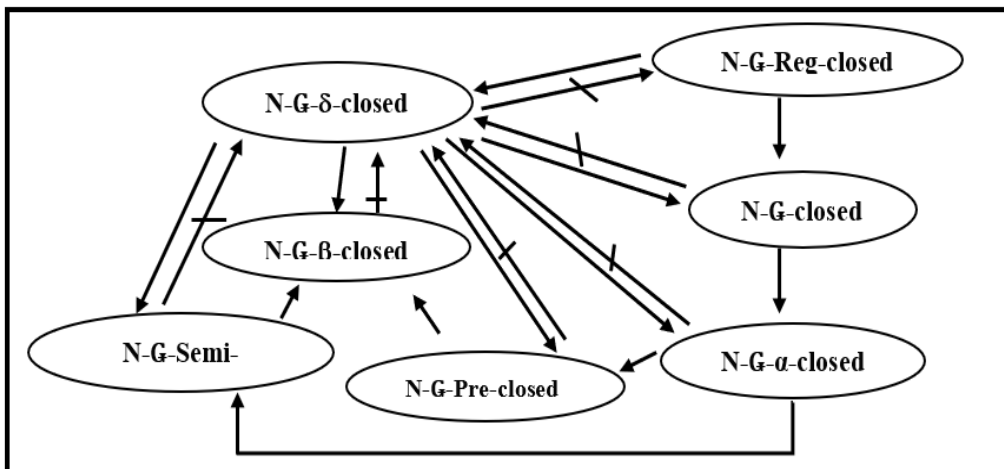


Diagram 3.2: Explain the Relation Between Some Types of N-G-Closed Sets

Definition 3.3

Let $(U, \mathcal{T}_{R(X)})$ be a Nano Grill Delta topological space and $F \subseteq U$. then, the Nano-G-

δ -interior (resp. Nano-G- δ -closure) of F is denoted by $Nint_{\delta G}(F) = \cup\{D: D \text{ is NG-Reg-open set and } D \subseteq F\}$ (resp. $Ncl_{\delta G}(F) = \cup\{X \in U: Nint_G(Ncl_G(D)) \cap F \neq \emptyset, D \text{ is N-G-open set and } X \in D\}$)

Remark 3.4

In NG-topological space, every NG-Reg- open set is NG- δ -open set but the converse not be true. We can show that by the following Example.

Example 3.5

Let $U = \{1, 2, 3, 4\}$, $U/R = \{\{1\}, \{3\}, \{2, 4\}\}$, $X = \{1, 2\}$.

Then, $\mathcal{T}_R(X) = \{U, \emptyset, \{1\}, \{2, 4\}, \{1, 2, 4\}\}$. Let $G = P(X) / \emptyset, A \notin G \Rightarrow A = \emptyset$.

$\Rightarrow \mathcal{T}_{R(X)G} = \{U, \emptyset, \{1\}, \{2, 4\}, \{1, 2, 4\}\}$.

Then, $\{1, 2, 4\}$ is NG- δ -open set. But, not NG-Reg- open set.

Theorem 3.6

If $(U, \mathcal{T}_{R(X)G})$ be a N-G- δ -topological space with respect to X where $X \subseteq U$ and F_1, F_2 are subsets of U then:

1. $Nint_{\delta G}(F_1) \cup Nint_{\delta G}(F_2) \subseteq Nint_{\delta G}(F_1 \cup F_2)$.
2. $Nint_{\delta G}(F_1) \cap Nint_{\delta G}(F_2) = Nint_{\delta G}(F_1 \cap F_2)$.

Theorem 3.7

Let $(U, \mathcal{T}_{R(X)G})$ be a N-G- δ -topological space with respect to X where $X \subseteq U$. let F_1 and $F_2 \subseteq U$ then:

1. $Ncl_{\delta G}(F_1 \cap F_2) \subseteq Ncl_{\delta G}(F_1) \cap Ncl_{\delta G}(F_2)$.
2. $Ncl_{\delta G}(F_1 \cup F_2) = Ncl_{\delta G}(F_1) \cup Ncl_{\delta G}(F_2)$.

Definition 3.8

Let $(U, \mathcal{T}_{R(X)G})$ be a Nano grill topological space, a subset F of $(U, \mathcal{T}_{R(X)G})$ is said to be:

1. Nano Grill Generalized closed set (N-G-g-closed) if $Ncl_G(F) \subseteq D$ where $F \subseteq D$ and D is N-G-open set in $(U, \mathcal{T}_{R(X)G})$.
2. N-G-g- δ -closed set if $Ncl_{\delta G}(F) \subseteq D$ where $F \subseteq D$, and D is N-G-open in $(U, \mathcal{T}_{R(X)G})$.
3. N-G-g-Reg-closed set if $N\text{-Reg-cl}_G(F) \subseteq D$ where $F \subseteq D$, and D is N-G-open in $(U, \mathcal{T}_{R(X)G})$.
4. N-G-g- α -closed set if $N\text{-}\alpha\text{-cl}(F) \subseteq D$ where $F \subseteq D$, and D is N-G-open in $(U, \mathcal{T}_{R(X)G})$.
5. N-G-g-semi-closed set if $N\text{-semi-cl}_G(F) \subseteq D$ where $F \subseteq D$, and D is N-G-open in $(U, \mathcal{T}_{R(X)G})$.
6. N-G-g-pre-closed set if $N\text{-pre-cl}_G(F) \subseteq D$ where $F \subseteq D$, and D is N-G-open in $(U, \mathcal{T}_{R(X)G})$.
7. N-G-g- β -closed set if $N\text{-}\beta\text{-cl}_G(F) \subseteq D$ where $F \subseteq D$, and D is N-G-open in $(U, \mathcal{T}_{R(X)G})$.

We can explain the relation between some types of N-g-closed sets.

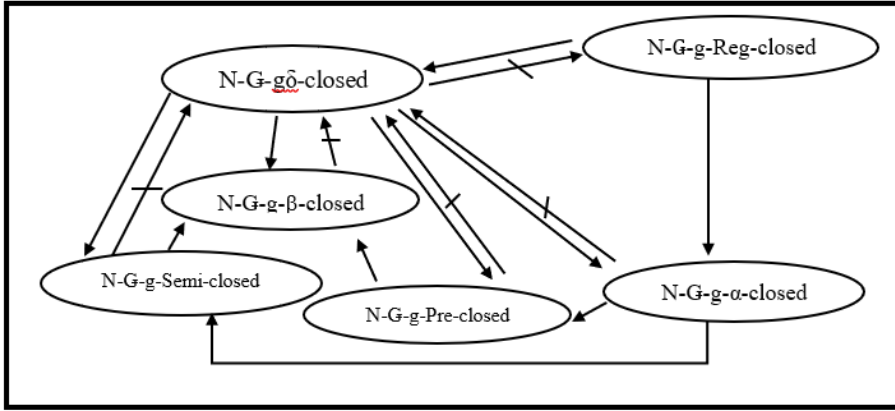


Diagram 3.3: Explain the Relation Between Some Types of N-G-G-Closed Sets.

Definition 3.9

Let $(U, \mathcal{T}_{R(X)G})$ be a Nano grill Generalized δ topological space and $F \subseteq U$, then F is said to be:

1. N-G-g- δ -Reg-closed set if $N-G-\delta-Reg-cl_G(F) \subseteq D$ where $F \subseteq D$, and D is N-G- δ -open in $(U, \mathcal{T}_{R(X)G})$.
2. N-G-g- δ - α -closed set if $N-G-\delta-\alpha-cl_G(F) \subseteq D$ where $F \subseteq D$, and D is N-G- δ -open in $(U, \mathcal{T}_{R(X)G})$.
3. N-G-g- δ -pre-closed set if $N-G-\delta-pre-cl_G(F) \subseteq D$ where $F \subseteq D$, and D is N-G- δ -open in $(U, \mathcal{T}_{R(X)G})$.
4. N-G-g- δ -semi-closed set if $N-G-\delta-semi-cl_G(F) \subseteq D$ where $F \subseteq D$, and D is N-G- δ -open in $(U, \mathcal{T}_{R(X)G})$.
5. N-G-g- δ - β -closed set if $N-G-\delta-\beta-cl_G(F) \subseteq D$ where $F \subseteq D$, and D is N-G- δ -open in $(U, \mathcal{T}_{R(X)G})$.

Remark 3.10

We can show the relation between some types of N-G-g- δ -closed set by the next diagram:

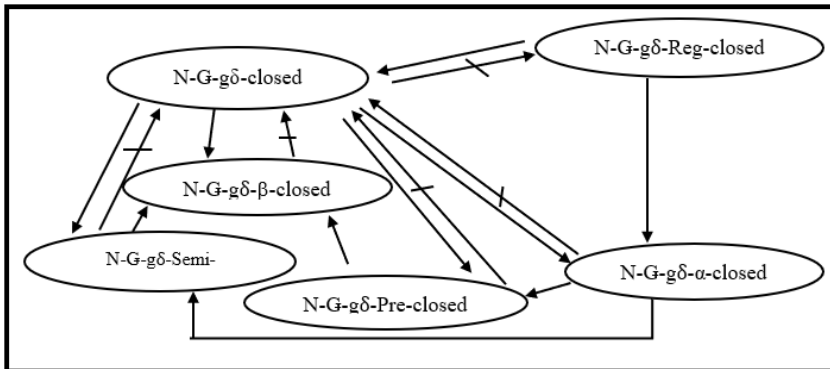


Diagram 3.4: The relation between some types of N-G-g δ -closed sets

Example 3.11

Let $U = \{1, 2, 3, 4\}$ with $U/R = \{\{1\}, \{3\}, \{2, 4\}\}$ and $X = \{1, 2\}$, then $T_R(X) = \{U, \emptyset, \{1\}, \{2, 4\}, \{1, 2, 4\}\}$, $T_R(X)^c = \{U, \emptyset, \{2, 3, 4\}, \{1, 3\}, \{3\}\}$ and $G = \{\{1\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}\}$, then:

$A - G = \{\{2\}, \{3\}, \{4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}\}$, $T_{R(X)G} = U - A$, $U \in T_{R(X)}$, $A \notin G$

$T_{R(X)G} = \{\emptyset, U, \{1\}, \{2\}, \{4\}, \{2, 4\}, \{1, 4\}, \{1, 3\}, \{1, 2\}, \{1, 2, 3\}, \{1, 2, 4\}\}$

$T_{R(X)G}^c = \{U, \emptyset, \{2, 3, 4\}, \{1, 3, 4\}, \{1, 2, 3\}, \{1, 3\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{4\}, \{3\}\}$

NG-g-closed = $\{U, \emptyset, \{1\}, \{2\}, \{4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}\}$

NG-g δ -closed = $\{U, \emptyset, \{2\}, \{4\}, \{1, 3\}, \{2, 4\}, \{1, 2, 3\}, \{1, 3, 4\}\}$

NG-g δ -Reg-closed = $\{U, \emptyset, \{2\}, \{4\}, \{1, 3\}, \{2, 4\}, \{1, 2, 3\}, \{1, 3, 4\}\}$

NG-g δ - α -closed = $\{U, \emptyset, \{1\}, \{2\}, \{4\}, \{1, 3\}, \{2, 4\}, \{1, 2, 3\}, \{1, 3, 4\}\}$

NG-g δ -Pre-closed = $\{U, \emptyset, \{2\}, \{4\}, \{1, 3\}, \{1, 4\}, \{2, 4\}, \{1, 2, 3\}, \{1, 3, 4\}\}$

NG-g δ -Semi-closed = $\{U, \emptyset, \{2\}, \{4\}, \{1, 3\}, \{2, 4\}, \{1, 2, 3\}, \{1, 3, 4\}\}$

NG-g δ - β -closed = $\{U, \emptyset, \{2\}, \{4\}, \{1, 3\}, \{2, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}\}$.

Definition 3.12

Let $(U, T_{R(X)G})$ be a Nano topological space and $V \subseteq U$ then V is said to be N-G-g- δ -Reg-open (resp. N-G-g- δ - α -open, N-G-g- δ -pre-open, N-G-g- δ -semi-open and N-G-g- δ - β -open) sets if its complement N-G-g- δ -Reg-closed (resp. N-G-g- δ - α -closed, N-G-g- δ -pre-closed, N-G-g- δ -semi-closed and N-G-g- δ - β -closed) sets.

Remark 3.13

We can explain the relation between some types of N-G-g- δ -open sets by the following diagram.

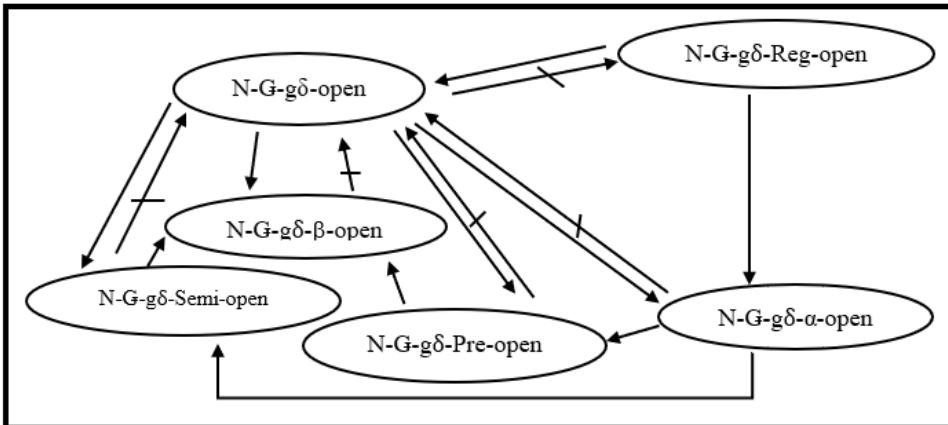


Diagram 3.5: The Relation Between Some Types of N-G-G δ -Open Sets.

Example 3.14: From example 2.10, we get:

NG-g-open = $\{\emptyset, U, \{2, 3, 4\}, \{1, 3, 4\}, \{1, 2, 3\}, \{3, 4\}, \{2, 4\}, \{2, 3\}, \{1, 3\}, \{4\}, \{3\}, \{2\}\}$

NG-g δ -open = $\{U, \emptyset, \{1, 3, 4\}, \{1, 2, 3\}, \{2, 4\}, \{1, 3\}, \{4\}, \{2\}\}$

NG-g δ -Reg-open = $\{U, \emptyset, \{1, 3, 4\}, \{1, 2, 3\}, \{2, 4\}, \{1, 3\}, \{4\}, \{2\}\}$

$NG-g\delta-\alpha\text{-open} = \{U, \emptyset, \{1\}, \{2\}, \{4\}, \{1, 3\}, \{2, 4\}, \{1, 2, 3\}, \{1, 3, 4\}\}$
 $NG-g\delta\text{-Pre-open} = \{U, \emptyset, \{1, 3, 4\}, \{1, 2, 3\}, \{2, 4\}, \{2, 3\}, \{1, 3\}, \{4\}, \{2\}\}$
 $NG-g\delta\text{-Semi-open} = \{U, \emptyset, \{2\}, \{4\}, \{1, 3\}, \{2, 4\}, \{1, 2, 3\}, \{1, 3, 4\}\}$
 $NG-g\delta-\beta\text{-open} = \{U, \emptyset, \{2\}, \{3\}, \{4\}, \{1, 3\}, \{2, 4\}, \{1, 2, 3\}, \{1, 3, 4\}\}.$

Remark 3.15

1. Every N-G-g- δ -Reg-open set is N-G-g- δ -open but the converse not necessary true.
2. Every N-G-g- δ -open set is N-G-g- δ - α -open but the converse not necessary true.
3. Every N-G-g- δ -open set is N-G-g- δ -pre-open but the converse not necessary true. the set $\{1, 2, 3\}$ is N-G-g- δ -pre-open set but not N-G-g- δ -open set.
4. Every N-G-g- δ -open set is N-G-g- δ -semi-open set but the converse not necessary true. The set $\{1, 2, 3\}$ is N-G-g- δ -semi-open set but not N-G-g- δ -open set.
5. Every N-G-g- δ -open set is N-G-g- δ - β -open set but the converse not necessary true. The set $\{1, 3, 4\}$ is N-G-g- δ - β -open set but not N-G-g- δ -open set.
6. Every N-G- δ -open set is N-G-g- δ -open set but the converse not necessary true. The set $\{2\}$ is N-G-g- δ -open set but not N-G- δ -open set.

Theorem 3.16

Every N-G- δ -closed set is N-G-g- δ -closed set.

Proof:

If F is N-G- δ -closed set then $N\delta cl_G(F) = F$. Therefore, every D is N-G- δ -open set such that $F \subseteq D$, $N\delta cl_G((F)) \subseteq D$ then F is N-G-g- δ -closed set.

Theorem 3.17

If $(U, \mathcal{T}_{R(x)})$ is Nano grill topological space. Then, F is N-G-g- δ -closed set in U . iff $N\delta cl_G(F) = F$ which is not have any non-empty N-G- δ -closed set.

Proof: Let F be N-G-g- δ -closed set and F_1 be N-G- δ -closed subset of $N\delta cl_G(F) - F$ then, $F \subseteq F_1^c$ and F_1^c is N-G- δ -open set. Since F is N-G-g- δ -closed set, $N\delta cl_G(F) \subseteq F_1^c$. Then, $F_1 \subseteq [N\delta cl_G((F))]^c$. Hence $F_1 \subseteq N\delta cl_G(F) \cap [N\delta cl_G((F))]^c = \emptyset$.

That is $F = \emptyset$, then $N\delta cl_G(F) - F$ not have any non-empty N-G- δ -closed set.

Conversely, Let $N\delta cl_G(F) - F$ have no non-empty N-G- δ -closed set and D be N-G- δ -open set such that $F \subseteq D$ that mean $N\delta cl_G(F) \cap D^c$ is N-G- δ -closed sub set of $N\delta cl_G(F) - F$, where $N\delta cl_G(F) \cap D^c \subseteq N\delta cl_G(F) \cap F^c$, where $F \subseteq D$ then $N\delta cl_G(F) \cap D^c = \emptyset$ and $N\delta cl_G(F) - F$ does not have any non-empty N-G- δ -closed set. Then, $N\delta cl_G(F) \subseteq D$. then, F is N-G-g- δ -closed set.

Conclusion

This paper presented and procured the concept of N-G-g- δ -closed set N-G-g- δ -open set via "Nano grill topological space". Then, established some new propositions and theorems with proving on "N-G-g- δ -TS". Further, furnished some counter examples where the result fails. We think our work can be can be studied and applied in soft topological spaces and in the search working via other areas of researches.

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