

NON-SPHERICAL EARTH EFFECTS ON THE GROUND TRACK SATELLITES AT DIFFERENT ALTITUDES

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Abstract: We address axisymmetric and stationary situations where there are perturbations of satellite orbits in the gravitational field of a body with a mass monopole and arbitrary spin multipole moments. The non-uniform mass distribution of the Earth causes gravitational forces for satellites operating in low-Earth orbits (LEO). All perturbation forces that affect the orbital elements also affect the satellite's ground track because it is connected to the orbital parameters. The importance of satellite systems in many facets of modern life has grown. By assessing various factors related to the use of the aforementioned systems and optimizing their benefits, the studies for achieving the intended aims can be realized. Placing the satellites in certain orbits that will deliver the appropriate features is crucial to achieving the anticipated aims. Another crucial task for the satellites is to maintain their orbits while maintaining their properties when they have reached such orbits. The perturbations influencing the satellites in their orbits are intended to achieve this goal, such as the harmonic coefficients J_n for the geopotential effect, must be accurately taken into account when choosing the appropriate orbits and locations. One of the most significant perturbation effects influencing the satellites' orbits and ground tracks is the J_2 effect, also known as the attraction flattening and brought on by the Earth's dynamic oblateness. The impact It is impossible to ignore the non-spherical gravitational perturbations on Earth. during the repeat-groundtrack orbit design process; in other words, Earth is not able to be regarded as an ideal sphere. The study reveals that disturbances can significantly alter satellite tracking, with a noticeable difference observed when satellites reach 1000 periods. The impact of disturbances on the satellites' ground track grows as the period of their orbit around Earth increases. The satellite's susceptibility to disturbance decreases with height and becomes more sensitive as the number of periods increases. The ground tracks appear more crunched as the satellites approach Earth's rotation. This is essentially due to the fact that we are getting closer to the ground track appears motionless because of the Earth's rotation. in a 24-hour orbit with respect to the Earth's surface.

Introduction

The two primary purposes of geodetic satellites are location in geodesy and monitoring the Earth's external gravitational field. Most positioning satellites orbit at a middle height of more than 1000 kilometers. The majority of satellites launched at 1000 kilometers altitude are intended for use in imaging and remote sensing applications. It is possible to retrieve the Earth's gravity field by satellites in low Earth orbit (1000 km or less). The perturbations in orbits are bigger than those in higher orbits because of their low altitudes. As such, they can be identified during orbit investigation to ascertain the gravitational field. In the case that the gravitational field

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of Earth resembles a central field, Kepler's first law states that the satellite's trajectory will be an ellipse around which the Earth is centered. The gravity field is centered in the first approximation. The center gravitational force component of the Earth's spherical shape, which propels the satellite into the Keplerian orbit, is then the main force maintaining the satellite in its orbit. The remaining forces, or perturbing forces, are causing the satellite to break out from its Keplerian orbit. Gravitational and nongravitational perturbing forces are the two main categories into which these forces are divided. It is evident that the Earth's gravitational perturbations have a stronger effect and, thus, cause larger and more reasonable perturbations on satellites that are closer to Earth. The difference between a satellite's actual orbit and the average Keplerian orbit that best fits the actual orbit is known as the perturbation [6].

Nowadays, one of the most crucial issues is figuring out how to compute an artificial satellite's orbit. In terms of calculation methods, the equations of motion are used with special perturbation techniques. These disturbances cause a drift for the satellite's orbital components. The satellite's height, which defines its angular velocity, and its location's latitude, which establishes the part of rotation that Earth has that applies there, are the two parameters that affect the ground track caused by Earth's rotation. When it comes to altering the trajectory that the earth would have taken if had remained stationary, the latitude and satellite altitude anywhere along the satellite's orbit will determine how this track is modified [1]. It is impossible to ignore the non-spherical gravitational perturbations on Earth. during the repeat-groundtrack orbit design process; in other words, Earth is not able to be regarded as a flawless sphere. A lot of earlier research (For example, Abramson, et al. [1]) simply took the disturbance of J_2 into account while designing and controlling the orbit, which is only appropriate for missions that don't require absolute precision [8]. Several renowned research on the repeat-groundtrack orbit have been published, with particular attention to the impact of high order disturbances. The beginning repeat-groundtrack orbit conditions for the high-order zonal perturbations of the gravitational field have been determined using an automated numerical approach. By considering the high-order tesseral terms of perturbations, Prior literature investigated, from an abstract perspective, what happens when equatorial periodic-orbit families split, producing repeat-groundtrack orbits [12]. The almost round Earth *repeat – groundtrack orbit* acquisition and satellite maintenance concerning the recurrence condition were demonstrated by Aorpimai and Palmer [2], Sengupta, et al. [17] used half-analytic methods to study the creation and upkeep of low eccentricity orbits for ground coverage; however, the design process only took the J_2 perturbation into account [17]. Based on a drift study over the whole groundtrack, Fu, et al. [7] investigated a strategy for maintaining a low Earth repetition groundtrack a series of reports; however, they used the two-body orbit as the reference orbit [7]. This study analyzes the ground track for the 100 periods and 1000 periods, and combines the effect of perturbation on the orbital elements into the computer model to produce the ground track of the satellite under the geopotential's harmonic coefficients J_n , up to $n = 6$.

Non-Spherical Gravity Potential and Perturbation Effect on Orbits

Non-Spherical Gravity Potential

A typical, idealized, or unperturbed motion can undergo perturbations. Numerical analysis is a highly appropriate tool for identifying disturbances. One of three main approaches—special perturbation techniques, general perturbation techniques, or

semi-analytic techniques can be used to examine the consequences of perturbations. Even though most are very small and are often disregarded, all of the forces that impact the satellite's orbit are recognized as perturbation forces, including drag, the third body, the accelerations produced by the central body, the pressure of solar radiation, and many more. It is now possible to investigate tide effects as a perturbation force because to advances in computational capacity [10]. A perturbation is a movement that deviates from a normal or expected path [4, 19]. The Keplerian path encircling the central entity will be disturbed by interactions with the space environment or by the gravitational pull of other celestial bodies, particularly nearby planets..

The following are the main unsettling forces that are present overall:

- ♦ gravitational forces brought about by the center body's non-spherical geometry and mass distribution.
- ♦ gravitational pull of other celestial bodies, including the planets, the Moon, and the Sun.
- ♦ accelerating force brought on by the pressure of solar radiation.
- ♦ acceleration force brought on by the residual atmosphere's drag [20].

One could classify these disturbances as either conservative or non-conservative. gravitational forces, including the attraction and pull of the Earth to other celestial bodies, are considered forces produced from potentials by conservatives. Forces that cannot be generated from a potential or dissipative forces involving energy loss are referred to as non-conservative forces. In addition to atmospheric drag, which drastically decreases with altitude, radiation pressure is the driving factor at work here [14].

Perturbation Effect on Orbits

Two parts are involved in how the gravitational field affects satellites. To begin with, the primary factor behind orbital motion is the gravitational pull. Precise orbit prediction depends critically on the modeling of the gravitational field. Second, a spaceborne gradiometer can measure the gravity gradients, which change depending on position and orientation [5, 15]. By definition, The body is called the geoid. that represents the surface to which the gravity vector (g) is always perpendicular, or the equipotential surface of the gravity potential at sea level. Geometrically speaking, the geoid is the Earth's equilibrium shape if all of its surface were covered in stagnant water. The masses of Earth are evenly distributed, and the geoid is a spherical in the zero-order approximation. Due to centrifugal forces, the rotation of the Earth causes the highest departure from the sphere by pushing the masses toward the equator. The radius of Earth is 6378.14 km at the equator, making it 21.4 kilometers more than it is at the poles. Consequently, subtracting a from the geoid sphere yields a rough oblate sphere that never veers away from the genuine oblate spheroid by more than 25 meters [20]. as shown in Figure 1.

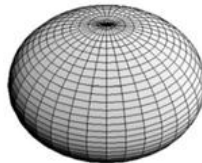


Figure 1: The Earth's Oblate Spherical Shape is Due to The Bulge the Equator [20]

The Earth's form and distribution of mass cause Distinctive square law impacts on an orbiting satellite, proving that it is not the gravitational source's point that The gravitational law of Newton anticipated. By using a sequence of the use of harmonics

that are spherical, which depict a *gravitational* object as a collection some more dominating than others in terms of mass centers, with the idea of a completely uniform sphere being the most common phrase, one can create an accurate earth model. The earth's gravitational potential can be written as [3, 21].

$$V = \frac{GM}{r} \left[1 + \sum_{n=1}^{\infty} \sum_{m=0}^n \left(\frac{a_e}{r} \right)^n (C_{nm} \cos(m\lambda) + S_{nm} \sin(m\lambda) P_{nm} \cos \phi) \right] \quad (1)$$

The harmonic coefficients C_{nm} , S_{nm} are mass integrals that depict the distribution of mass inside the central body; these include the equatorial radius and P_{nm} otherwise known as related Legendre functions or Legendre polynomials. Since the initial component, GM / r , relates to Keplerian motion and explains the potential of a homogeneous sphere, it is also known as the Keplerian term. The remaining expressions in the double-summation are caused by the disruptive potential R ; as a result, the terms where $m = 0$ and $n = 1$ are reduced to zero and the coordinate system's origin is moved to the primary's center of mass [3, 16].

$$R = \frac{GM}{r} \left[\sum_{n=2}^{\infty} \sum_{m=0}^n \left(\frac{a_e}{r} \right)^n (C_{nm} \cos(m\lambda) + S_{nm} \sin(m\lambda) P_{nm} \cos \phi) \right] \quad (2)$$

The term $R = \frac{GM}{r}$ represents the gravitational potential R at a distance r from a mass M , where G is the universal gravitational constant. An orbiting body's different perturbative influences can be taken into account via a disturbing function that expresses the gravitational potential function. N is the degree, m is the order, $\frac{a_e}{r}$ indicates a normalized radius (where a_e could represent Earth's mean radius), C_{nm} , S_{nm} are coefficients that depend on specific physical properties or measurements, λ represents longitude, ϕ represents latitude. The complete disturbing function R into equations (2) incorporates contributions from multiple sources, including Non-spherical harmonics of the gravitational field experienced by Earth [18]. These spherical harmonics can be classified in three groups:

•**Zonal Harmonics:** The zeroth order ($m = 0$) defines them. This suggests that the potential's reliance on longitude (λ) disappears and that the field is symmetrical at the polar axis, with $n+1$ horizontal zones appearing to divide the Earth. Because of the curvature of the Earth, J_2 is by far the strongest perturbation [13].

•**Sectorial Harmonics:** They happen when the degree ($n=m$) equals the harmonic's order.

When this occurs, Earth is split into two n meridians sections, and the polynomials are zero at the poles [13]

•**Tesseral Harmonics:** For cases in which $n \neq m \neq 0$, The tesseral harmonics make an effort to simulate particular Earth locations that deviate from a perfect sphere [13].

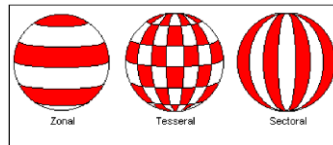


Figure 2: Types of Spherical Harmonics [24].

Using a Ground Tracking Satellite to Determine the Earth's Gravity Field and Definition of Satellite Ground Track

Using a Ground Tracking Satellite to Determine the Earth's Gravity Field

A ground tracking satellite may determine satellite laser ranging (SLR), Doppler orbitography, and radiopositioning integrated by satellite methods to measure the Earth's gravitational field (*DORIS*), and precision equipment with range and range-rate (*PRARE*). When employing a ground tracking satellite, the primary observed quantities are the tracking station's direction, range, range rate, and phase with respect to the satellite being observed. The satellite orbit and the ground tracking station can be connected via geometric and physical functional linkages that can be ascertained using these data from observations. The Earth's gravity field may be calculated because the satellite orbit is the implicit function of the perturbation factors of the Earth's gravity field [9].

Satellite Ground Tracks

A satellite's ground track is the collection of locations where the position vector r crosses the planet's surface. The ground track of an Earth-orbiting satellite is the orbit's projection onto the planet's revolving surface. Ground traces are obviously helpful because they display the precise geographic areas over which the satellite passes. Figure (3) shows a ground track for an inclined geocentric satellite.

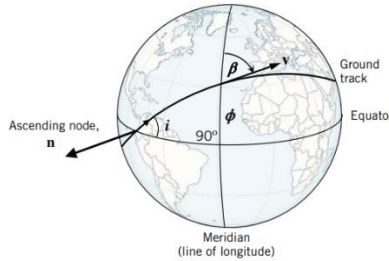


Figure 3: Satellite Ground Track [11].

Spacecraft orbit and perigee were specified by the three Euler angles, ψ , θ , and χ . In this instance, we would like to identify S . We derive the relationship between the orbital elements and the Euler angles. using Figure 4:

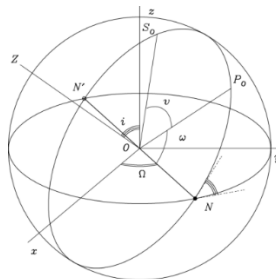


Figure 4: Ground Track and Orbital Elements [14].

$$\psi = \Omega \quad (3)$$

$$\theta = i \quad (4)$$

$$\chi = \omega + v \quad (5)$$

The angles Ω , ω , and $M - nt$ For a true orbit, they are fixed, while for a Keplerian orbit, they change with time. The inclination i doesn't change. The separation between S and the attraction's center O is provided by (6)

$$r = r(\theta) = \frac{a(1 - e^2)}{1 + e \cos(\theta - \theta_0)} \quad (6)$$

where (r) The radial distance from the focus to the orbiting body at a given angle θ , a: The semi-major axis of the conic section. e: The eccentricity of the conic section, θ_0 : The angle that defines the orientation of the ellipse in relation to a reference direction. stated with relation to the true anomaly v :

$$r = \frac{a(1 - e^2)}{1 + e \cos v} \quad (7)$$

It can be demonstrated that any solid rotation can be broken down into three simple rotations around appropriately selected axes. This decomposition is one-to-one with the of the Euler anglesthe following domains [14]

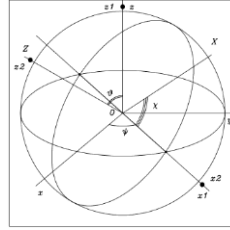


Figure 5: The Three Rotations That Use The Three Euler Angles to Transfer A Particular Sphere Point to an Arbitrary New Location. The Three Rotational Axes Are Denoted by Black Circles. $N: O_z = O_{z1}$ for $[P_1]$, $O_{x1} = O_{x2}$ for $[P_2]$, and $O_{z2} = O_z$ for $[P_3]$

$$\psi \in [0, 2\pi), \quad \theta \in [0, \pi), \quad \chi \in [0, 2\pi)$$

Here is a summary of the rotation's axes and angles.:

$$\begin{aligned} (Ox, Oy, Oz) &\xrightarrow{P_1} (Ox_1, Oy_1, Oz_1 = Oz) \\ (Ox_1, Oy_1, Oz_1) &\xrightarrow{P_2} (Ox_2 = Ox_1, Oy_2, Oz_2) \\ (Ox_2, Oy_2, Oz_2) &\xrightarrow{P_3} (OX, OY, OZ = Oz_2) \end{aligned}$$

And lastly, the three matrices of rotation:

$$P_1 = \begin{pmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (8)$$

$$P_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{pmatrix} \quad (9)$$

$$P_3 = \begin{pmatrix} \cos \chi & -\sin \chi & 0 \\ \sin \chi & \cos \chi & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (10)$$

These three matrices' matrix product yields the matrix P that is computed below. We assume that N is at the time origin on the axis Ox without losing generality. So, (r, 0, 0) are its coordinates. By applying P to the coordinates of N(x, y, z), the coordinates of S(X, Y, Z) are obtained.

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = P \begin{pmatrix} x \\ y \\ z \end{pmatrix} = P \begin{pmatrix} r \\ 0 \\ 0 \end{pmatrix}$$

It is evident that this computation will only need the matrix P's initial column. Thus, we will compute the product of the matrix. $P = P_1 P_2 P_3$ Then fill out the form with it.

$$P = \begin{pmatrix} \cos \psi \cos \chi - \sin \psi \sin \chi \cos \theta & P_{12} & P_{13} \\ \sin \psi \cos \chi + \cos \psi \sin \chi \cos \theta & P_{22} & P_{23} \\ \sin \chi \sin \theta & P_{32} & P_{33} \end{pmatrix} \quad (11)$$

which gives

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = r \begin{pmatrix} \cos \psi \cos \chi - \sin \psi \sin \chi \cos \theta \\ \sin \psi \cos \chi + \cos \psi \sin \chi \cos \theta \\ \sin \chi \sin \theta \end{pmatrix} \quad (12)$$

Using the provided orbital parameters by (3) – (7), we obtain

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \frac{a(1 - e^2)}{1 + e \cos v} \begin{pmatrix} \cos \Omega \cos(\omega + v) - \sin \Omega \sin(\omega + v) \cos i \\ \sin \Omega \cos(\omega + v) + \cos \Omega \sin(\omega + v) \cos i \\ \sin(\omega + v) \sin i \end{pmatrix} \quad (13)$$

Now let us consider a spherical coordinate system in a Galilean frame. $\mathfrak{R}$. The equatorial plane xO_y of the Earth serves as the plane of reference, the polar axis is represented by O_z , and O_x 's position is fixed in space. The point (S) can be specified in $\mathfrak{R}$ by its spherical coordinates, the longitude λ and the latitude ϕ , measured using the standard method that derives from the system of right-handed trigonometry. (The longitude of O_x (location of N at the time origin) is indicated by λ_0) Hence

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = r \begin{pmatrix} \cos \phi \cos(\lambda - \lambda_0) \\ \cos \phi \sin(\lambda - \lambda_0) \\ \sin \phi \end{pmatrix} \quad (14)$$

As a result, we are given S position. (λ, ϕ) according to time and the other orbital

parameters via X, Y, Z :

$$\phi = \arcsin \frac{Z}{r} \quad , \quad \lambda = \lambda_0 + \arccos \frac{X}{r \cos \phi} \quad (15)$$

$$\lambda - \lambda_0 \quad \text{from the sing of } Y \quad , \quad \lambda - \lambda_0 \in (-\pi, +\pi] \quad (16)$$

Results and Discussion

We rarely take the time to construct a complete 3D model when analyzing the geometry and physical configuration of an orbit with regard to the earth. A ground track is a more effective method to visualize this relationship. A ground track is a two-dimensional map of Earth that displays a spacecraft's trajectory over the planet's surface. We are aware of Earth's rotation. The ground tracks will now undergo several changes. We will obtain a great deal of information regarding the orbit, but this will also add complication. Nodal displacement is the first variable we will define. As everyone may remember, the ascending node point is the point at which the satellite crosses over from the southern to the northern hemisphere. This region is created by the rotation of the Earth. The longitudinal distance between the ascending nodes can be accustomed to determine the orbit's semi-major axis. the distance between the start and finish points.

Table 1: Data Orbit Elements for Different Altitudes Satellites

orbit	h(km)	a(km)	rp	ra	e	I(deg)	Ω (deg)	Ω (deg)
Low	350	6728	6683.5	6772.5	0.006614	40	112.2	4.6
	608	6987	6940.8	7033.2	0.006614	40	112.2	4.6
	859	7237	7189.1	7284.9	0.006614	40	112.2	4.6
	5891	12270	1218.9	1235.1	0.006614	40	112.2	4.6
medium	8065	14444	1434.8	1454.0	0.006614	40	112.2	4.6
	10390	16768	1665.7	1687.9	0.006614	40	112.2	4.6
	19129	25507	2533.8	2567.6	0.006614	40	112.2	4.6
high	23615	29993	2979.5	3019.1	0.006614	40	112.2	4.6
	35785	42163	4188.4	4244.2	0.006614	40	112.2	4.6

The latitudinal distance from the equator to the maximum or minimum of the *ground track* is known as the *inclination*. As can be seen in the figures (5) TO (13), the maximum latitude that the satellites were able to reach was approximately 40 degrees, which corresponds to the input data's 40° inclination. An elliptical orbit will not be symmetric, meaning that it will be more compressed or "scrunched" in certain locations and more stretched in others also the ground track areas where the satellite is traveling at a slower speed will appear "shorter" than the sections where it is traveling at a faster speed. Since the satellites employed in this part have very minor eccentricities, their orbital ground tracks will be more uniform, as seen in Figure 5 to Figure 13 , Table 1 displays the data for ground track, which consists satellites at three different altitudes.

The study analyzes the ground track for the 100 periods and 1000 periods, as shown in figures, and combines the effect of perturbation on the orbital elements into the computer model to produce the ground track of the satellite under these forces. Figure 5, Figure 6 and Figure 7 represent the obtained ground truck of a low-Earth orbit satellite under the assumptions of a Keplerian orbit and a perturbed orbit for the

satellite's first cycle throughout the 100 and 1000 period orbiting. It is evident how the perturbation forces cause a difference. is subjected to quickly shifting conditions as it travels through a variety of locations and local time zones, all of which have a substantial impact on the potential perturbation of Earth. Determining the satellite's ground track is linked to investigating how this disturbance alters the satellite.

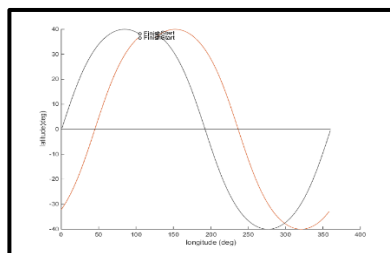
Because a satellite's position on Earth changes significantly with an increase in its number of rotations around the planet, the data clearly illustrate the extent to which disturbances can affect satellite tracking. The results show that there is a noticeable difference in tracking when the satellites reach 1000 periods. The Figures 8, Figure 9 and Figure 10 illustrate mid-orbit satellites that were included in the study. It is observed that disturbance has some effect on the satellites' ground track, and that this impact grows as the period of the satellites' around the Earth rises. The effect of the gravitational perturbation on high-altitude satellites as they complete 100 and 1000 rotations are shown in Figure 11, Figure 12 and Figure 13; however, it should be noted that the susceptibility of the satellite to the disturbance decreases with increasing height. It is also observed that the satellite becomes more sensitive to perturbations as the number of periods increases. The ground tracks appear more scrunched the longer the periods get. This is essentially due to the fact that we are getting closer to the rotation of the Earth, which creates the ground track appear stationary in a 24-hour orbit with relation to Earth's surface Figure 13.

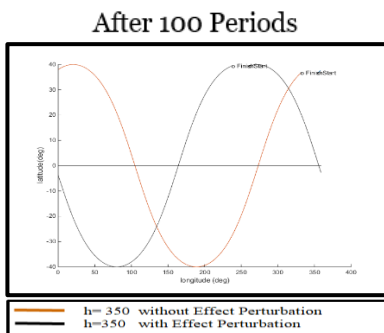
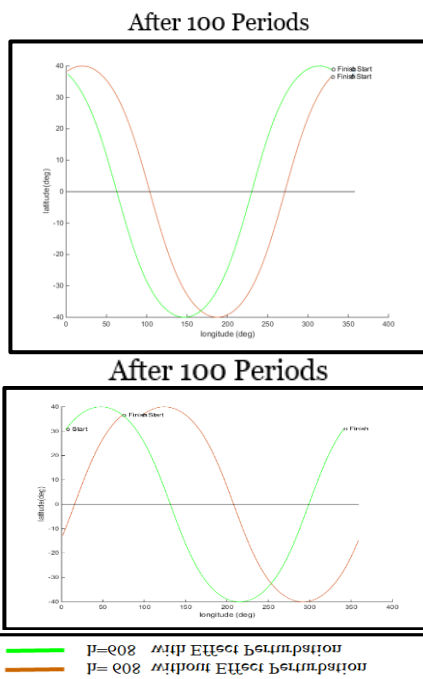
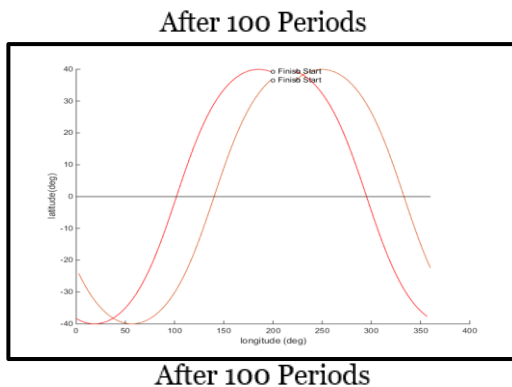
Conclusion

The results shows that if the computation is done early in the satellite's orbital motion, the impact of orbit perturbations on the Earth potential applied to a satellite in high Earth orbit can be omitted. However, this effect takes on significant values in for a long time calculations, therefore it must be taken into consideration. For low and high periods, anyway, the effects on the low and mid orbits must be taken into account. Because a satellite's position on Earth changes significantly with an increase in its number of rotations around the Earth, the data clearly illustrate the extent to which disturbances can affect satellite tracking. The results show that there is a noticeable difference in tracking when the satellites reach 1000 periods.

This asymmetry leads to variations in the ground track of the satellite as it orbits the Earth. In regions where the orbit is more compressed, the satellite will travel at a slower speed, causing those sections of the ground track to appear shorter. The eccentricity of satellites which is used in this thesis have a minor eccentricity will have more uniform orbital ground tracks due to their nearly circular orbits. Additionally, as the number of orbital periods increases, satellites become more sensitive to perturbations. This means that over time, small disturbances can have a greater impact on the satellite's trajectory and position.

After 100 Periods



**Figure 5:** Ground Tracks of Low Orbit Satellite.**Figure 6:** Ground Tracks of Low Orbit Satellite.

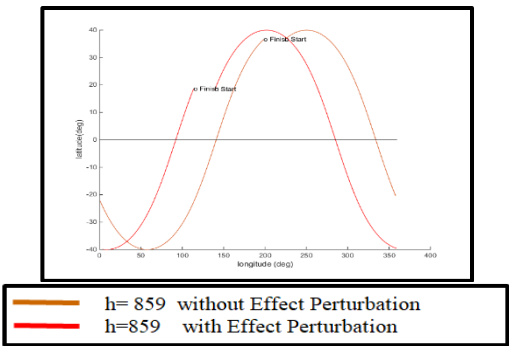


Figure 7: Ground Tracks of Low Orbit Satellite.

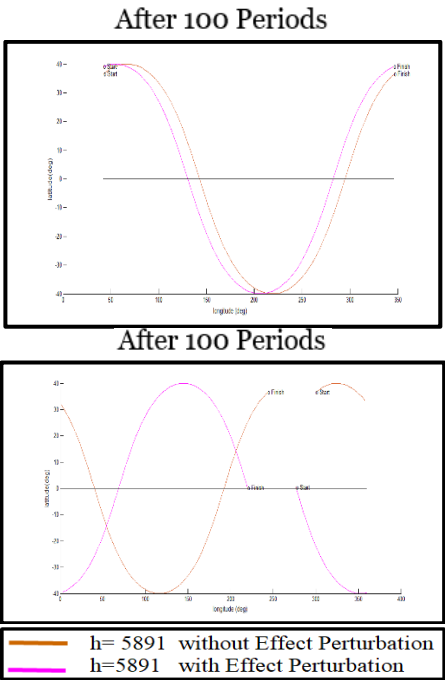
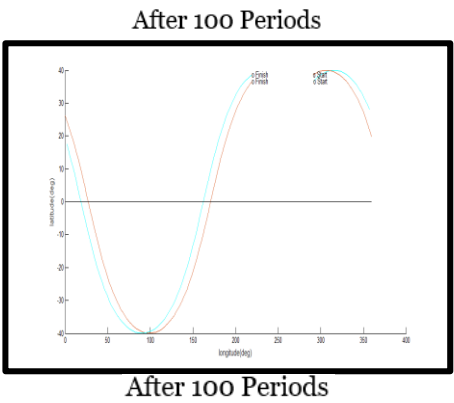


Figure 8: Ground Tracks of Medium Orbit Satellite.



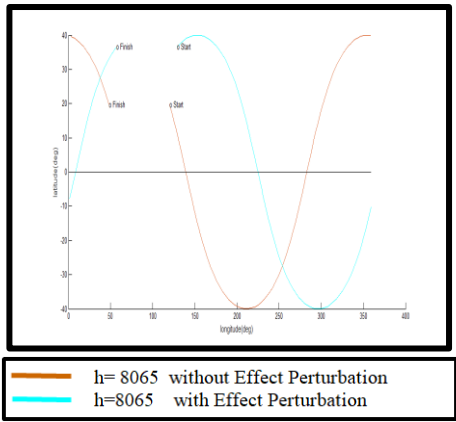


Figure 9: Ground Tracks of Medium Orbit Satellite.

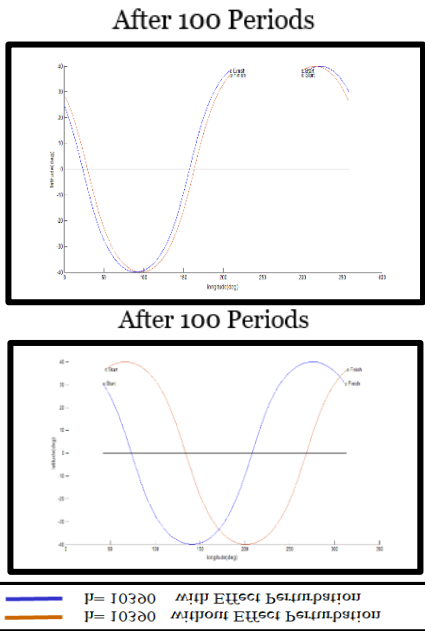
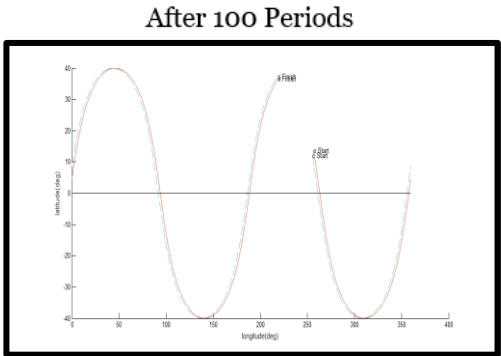


Figure 10: Ground Tracks of Medium Orbit Satellite.



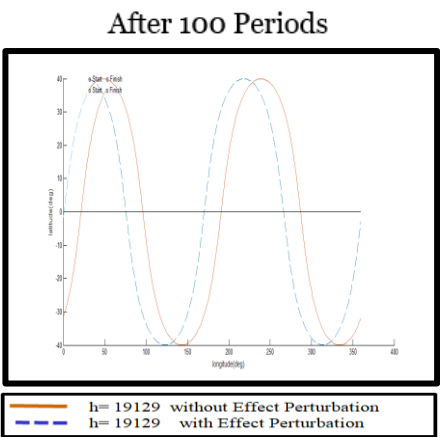
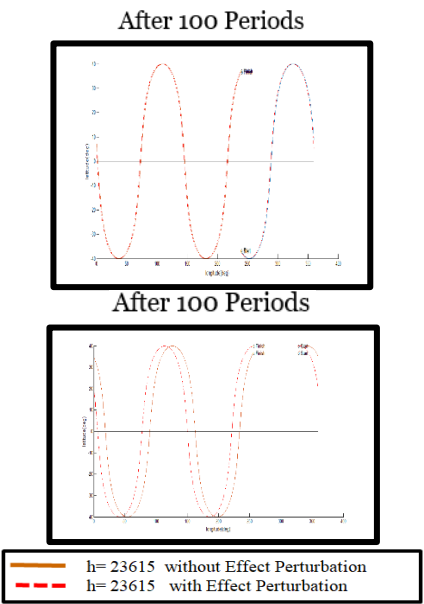
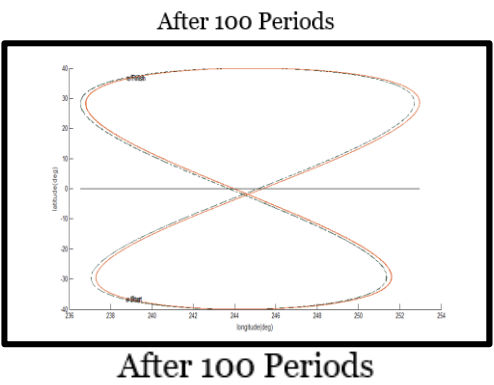


Figure 11: Ground Tracks of High Orbit Satellite.



Figures 12: Ground Tracks of High Orbit Satellite.



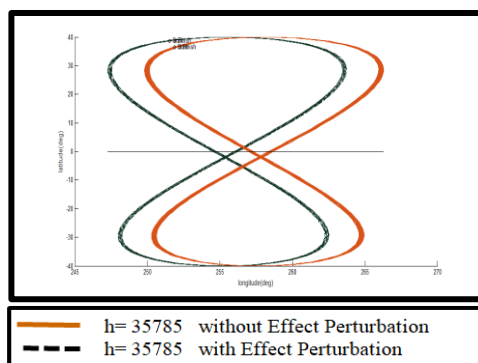


Figure 13: Ground Tracks of High Orbit Satellite.

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