

ANALYZING THE EFFECT OF NONSPHERICAL EARTH ON SATELLITES ORBITAL ELEMENTS AT DIFFERENT ALTITUDES.

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Abstract: The gravitational field of Earth is not entirely isotropic; rather, its values change dramatically with one's orientation. Changes in its interior density and non-spherical shape are the reason. These gravitational anisotropies affect a spacecraft's orbit around the Earth, causing slightly distorted Keplerian orbits. When we get close enough to track a satellite's motion, we may see that it deviates from this Keplerian motion. Throughout a single revolution, the difference is negligible; still, it gradually grows over time, becoming noticeable after a few days. It is possible to describe the Keplerian orbit as gradually deformed. Orbital elements are what distinguish a certain orbit in celestial mechanics. These elements are required to describe the motion of celestial bodies in space as well as to predict and understand their trajectories. Artificial satellite orbit theory about the study of the Earth's external gravitational field, the goal of satellite orbit theory is to establish a relationship between orbital parameters and gravitational field features, like coefficients, using a description expressed in terms of spherical harmonics. When tracking satellites, the physical effects resulting from the zonal harmonics are crucial because they show how the earth's actual shape differs in latitude from the symmetric earth. This is due to the fact that the earth's density might differ, either higher or lower, at one latitude point compared to another. The orbit of a test satellite is integrated over a comparatively short period of one day using the celestial mechanics program. Eccentricity, inclination, and semi-major axes all have periodic effects; however, the other three factors show a secular change in addition to brief periodic variations. Based on the results, we may conclude that as the satellite's altitude increases, the gravitational potential's impacts on its orbital elements decrease. The amount to which variations in the satellite's altitude impact osculating Keplerian compounds in radial, along-track, and out-of-plane directions relative to the starting orbit were also examined in the study. The results showed that the degree to which a satellite influences gravitational potential depends critically on its elevation. We infer that the mass distribution is not precisely spherically symmetrical and because there is a non-spherical discrepancy between the actual motion and the ideal Keplerian motion affected by the Earth.

INTRODUCTION

This work introduces a new set of orbital elements to fully represent the zonal harmonics problem around an oblate celestial body. Earth is seen as a spherical mass with homogeneous density according to the Kepler issue. Although this technique provides an adequate approximation or "first guess" of a satellite's motion, the effects of disturbances have been ignored. A satellite will depart from Kepler motion due to several primary perturbations, including thrust, air drag, solar radiation pressure, non-spherical earth gravitational forces, and third-body gravitational effects. Among these changes, the non-spherical earth contributions are significant for near-earth satellites. As is assumed in the Kepler problem, the

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earth is not spherical and does not have a uniform distribution of density. Secular, long-period, and short-period changes in a satellite's motion are caused by these variations. According to secular variations, an element would either monotonically rise or decrease from its starting values. Therefore, long-period variations result from the presence of sinusoidal terms with arguments containing slowly varying elements, whereas short-period variations result from sinusoidal terms with the fast element in the argument. In contrast, periodic variations produce element values that oscillate about the initial element values [5]. However, this method of treating the mass of the bodies is no longer applicable when considering the impact of perturbations on the orbit of an Earth satellite. The earth is asymmetrical, flattening in the poles and bulging at the equator instead of being a spherically symmetric mass. Due to forces originating from the earth's gravity field, this uneven mass distribution causes the most prominent perturbation on a near-Earth satellite [1].

The bulk of satellites used for positioning are in medium orbits, with altitudes exceeding 1000 km. Remote sensing and imaging are the primary goals of most satellites launched at 1000 km altitude. Satellites orbiting at a low altitude of 1000 km or less are appropriate for retrieving the Earth's gravity field. The orbital disturbances are bigger than those in higher orbits due to their low heights. As such, they can be identified during orbit analysis to determine the gravitational field, Kepler's first law states that if the Earth's gravitational field resembles a central field, the satellite's trajectory will be an ellipse on which the Earth is focused. Firstly, the gravitational field is central in the approximation. The satellite enters the Keplerian orbit due to the spherical Earth, and this forces it to remain in orbit mostly because of the center gravitational force component. The remaining forces, or perturbing forces, are causing the satellite to break out from its Keplerian orbit. These forces can be divided into two main categories: affecting gravitational forces and those that are not [4, 7, 9, 13, 17]. The Earth's gravitational perturbing effects have a stronger.

On closer satellites, it is evident that the Earth's gravitational perturbations have a stronger effect and cause larger, more logical perturbations. As the difference between a satellite's actual orbit and the average Keplerian orbit that best fits the actual orbit, the perturbation is defined as such. An initial accurate assessment of the real orbit is necessary for an accurate assessment of a perturbation [6]. With their studies, publications, and contributions to science, these researchers have significantly advanced the topic of zonal harmonics, prior research on zonal harmonics, mathematical functions that represent the Earth's gravitational field, discovered that they are critical for accurately modeling the Earth's gravitational field in geodetic applications such as satellite orbit determination and understanding sea level variations, emphasizing the significance of higher-order zonal harmonics. Montenbruck and Gill [15] explores various aspects of satellite orbits, including the influence of non-spherical gravity fields on satellite dynamics, the authors discuss practical applications of zonal harmonics in orbit determination and analysis, providing insights into how these effects are accounted for in satellite missions [15], Kaula [12] research on zonal harmonic disturbances of Earth's gravitational field explores their implications for geodesy and geophysics. These disturbances, characterized by differences in mass distribution, deviate from Earth's spherical shape due to changes in gravitational field strength [12]. Prior study on geomagnetic secular variation found that perturbations in zonal harmonics can significantly impact geomagnetic processes. These disturbances, such as solar wind, internal dynamo activities, and crustal movements, alter the amplitudes and phases of zonal harmonics, impacting applications such as power grids, navigation systems, and space weather forecasts. A study on the impact of gravitational pull, zonal harmonic

perturbations, and tidal interactions on Earth's rotation and geophysical processes revealed the significant influence of disturbances. The study conducted by Izzet, et al. [11] examined the impact of perturbations from the moon and sun on Earth satellite orbital elements [11], Izzet, et al.'s determination of the Earth potential disturbance on satellites in low-Earth orbit, for both prograde and retrograde trajectories [10]. In this paper, we have studied the motion of satellites at different altitudes under the influence of the oblateness of Earth. We have analyzed results for 1 day the orbital elements have been computed for the above-mentioned period, We note that eccentricity, tilt, and semi-major axes all have periodic effects; The other three factors are: the right rise of the ascending node, argument, and The mean anomaly show secular change in addition to brief periodic variations, Based on the results, we may conclude that as the satellite's altitude increases, the gravitational potential's impacts on its orbital elements decrease .The amount to which variations in the satellite's altitude impact osculating Keplerian compounds with relation to the original orbit in radial, along-track, and out-of-plane directions.was also examined in the study. The results showed that the degree to which a satellite influences gravitational potential depends critically on its elevation.

DISTURBANCES

Disturbances are the departures from a standard, idealized, or undisturbed motion. Most of the time, we assume that the cosmos is quite regular and predictable. The real motion will not match the ideal two-body path because of disturbances from additional bodies (As the Moon and Sun, for example) and additional forces (like drag and a non-spherical core body) that are not considered in Keplerian motion. Note that to show that these perturbation techniques may be used to any central body, we refer to the central body [10], The most important perturbations are classified into four main groups:

- Gravity of a non-spherical body: The point-mass model is erroneous due to the non-spherical shape of the major body (the Earth, the Sun, or the body under consideration).
- Third-body perturbations: A gravitational force that we overlooked in the two-body problem-solving is exerted by the Sun, Moon, and other planets in our solar system.
- Atmospheric drag: The atmosphere's force acts against the body's direction of travel [8].
- Pressure from solar radiation: The force generated by the sun's light's incidence on the body is called solar radiation pressure [2, 14].

Earth's Orbital Elements One might make the comparison between Earth seen as a satellite of the Sun and an artificial satellite in orbit around the Earth. However, let us first highlight a fundamental distinction between the two issues:

- The man-made satellite is viewed as a point about Earth, and the flattening of the attracting body mostly disturbs its Keplerian motion (through the Earth's J_2 term).
- Due to its status as a satellite of the Sun the Earth cannot be understood as a point, but rather as a rotating solid with a non-spherical J_2 term that generates perturbations and a polar axis of rotation that determines the equatorial plane. The semi-major axis of the Earth's orbit is (a), and its eccentricity is (e). These are the elements of the Earth's Keplerian orbit The obliquity, or the angle formed by the Earth's equatorial plane and its orbital plane, is represented here by (i). The angle formed by an arbitrary star-fixed origin and a line of nodes, or the

direction of the vernal point and the line of the equinoxes, is represented by (Ω). The angle formed by the vernal point and perihelion's directions is represented by (ω). The obliquity, or the angle formed by the Earth's equatorial plane and its orbital plane, is represented here by (i). The angle formed by an arbitrary star-fixed origin and a line of nodes, or the direction of the vernal point and the line of the equinoxes, is represented by (Ω). The angle formed by the vernal point and perihelion's directions is represented by (ω) [3]. There are two reasons why the ideal Keplerian motion differs from the true motion:

- Both the mass distribution and the Earth are not precisely spherically symmetric.
- In addition to the pull of Earth, the satellite senses forces that are generally categorized as frictional and attractive forces resulting from other celestial bodies. [3].

Gravitational Potentials

Small bodies are primarily distinguished by their irregular forms, which result in substantially non-spherical mass distributions. Close to these bodies, this trait causes a lot of intriguing interactions. indicates that A massless particle with a randomly distributed mass is an object of attraction. First, we integrate the differential mass element's potential over the whole body to set up the potential for this case [3, 16].

$$U(r) = \mathcal{G} \int_B \frac{dm(\rho)}{|r - \rho|} \quad (3.1)$$

Figure 1 illustrates the geometry of the mass distribution, where ρ is the position vector of the differential mass element dm , B is the collection of all mass elements, and $\mathcal{G}$ is the gravitational constant. It can be demonstrated that the gravitational potential satisfies both Poisson's equation (with σ standing for local density) and Laplace's equation (with $\nabla^2 U = 0$ outside the body and $\nabla^2 U = -4\pi\mathcal{G}\sigma$ inside the body). Integration of this potential for a sphere with constant density, total mass M , and radius R produces:

$$U = \begin{cases} \frac{\mathcal{G} \cdot M}{|r|} & r > R \\ \frac{\mathcal{G} \cdot M}{2R^3} (3R^2 - r^2) & r \leq R \end{cases} \quad (3.2)$$

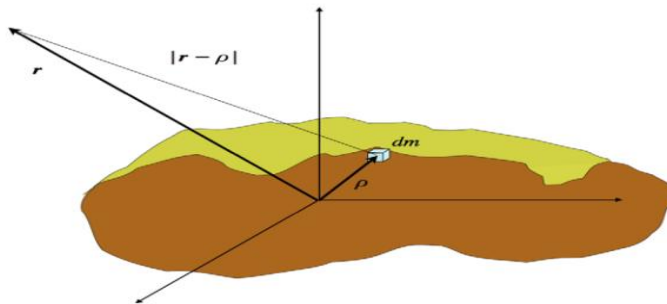


Figure 1: Mass distribution geometry [3].

Spherical Harmonics Models

A nonspherical body's gravitational potential can be specified in several ways. Any mathematical solution to Laplace's equation that matches the distribution of

physical masses will be valid. Making a collection of orthogonal Laplace equation solutions and selecting the expansions' coefficients to correspond with the actual potential function is a wonderful way to take advantage of this phenomenon. Separating the variables in terms of spherical coordinates is one way to specifically solve Laplace's equation [12].

Indicate which spherical coordinates belong to a given position vector. $r = x.\hat{x} + y.\hat{y} + z.\hat{z}$ as:

$$r = \sqrt{x^2 + y^2 + z^2} \quad (4.1)$$

$$\sin \delta = \frac{z}{r} \quad (4.2)$$

$$\tan \lambda = \frac{y}{x} \quad (4.3)$$

where δ is the latitude and λ is the longitude. The general form for the spherical harmonic potential for a gravity field is then:

$$U.(r, \delta, \lambda) = \frac{\mu}{r} \sum_{l=0}^{\infty} \sum_{m=0}^l \left(\frac{r_o}{r}\right)^l P_{lm}(\sin \delta) [C_{lm} \cos m\lambda + S_{lm} \sin m\lambda] \quad (4.4)$$

where r_o is the normalizing radius (often selected as the body's mean or maximum radius), P_{lm} stands for the Associated Legendre Functions, and C_{lm} and S_{lm} are referred to as the gravity field harmonic coefficients (also known as Stokes coefficients). where $\mu = \mathbf{gM}$ is the gravitational parameter of the body. Determining these coefficients' specifications is similar to figuring out the body's potential and mass distribution. The closed-form relationship can be used to define the Associated Legendre Functions [12, 15].

$$P_{lm}(\sin \delta) = \cos^m \delta \sum_{i=0}^{\text{int}[(l-m)/2]} T_{lmi} \sin^{l-m-2i} \delta \quad (4.5)$$

$$T_{lmi} = \frac{(-1)^i (2l-2i)!}{2^l i! (l-i)! (l-m-2i)!} \quad (4.6)$$

where the integer portion of x is returned by the $\text{int}[x]$ function. One crucial characteristic of the spherical harmonic expansion is functional orthogonality. Determine the amount S_{lmi} for $i = 0, 1$ as

$$S_{lm0} = C_{lm} P_{lm}(\sin \delta) \cos(m\lambda) \quad (4.7)$$

$$S_{lm1} = S_{lm} P_{lm}(\sin \delta) \sin(m\lambda) \quad (4.8)$$

The easiest way to express the orthogonality property is as follows:

$$\int_{\delta=-\pi/2}^{\pi/2} \int_{\lambda=0}^{2\pi} S_{lmi} S_{h kj} \cos \delta \, d\delta \, d\lambda = \frac{4\pi(l+m)! \delta_i^j \delta_m^k \delta_h^l}{(l-m)! (2l+1)(2-\delta_m^0)} \quad (4.9)$$

Where δ_i^j is the Kronecker delta function, and its value is equal to 1 if $i = j$ and 0 otherwise. As a result, the integral is only non-zero in the event that the three indices coincide. The orthogonality relations can be used to directly derive the gravity

coefficients C_{lm} and S_{lm} given a mass distribution. In particular, suppose that the body's density is given at every location as $\sigma(\rho)$. From the integrals, the gravity coefficients may then be determined [15]:

$$(C, S)_{lm} = \frac{(2 - \delta_m^0)}{M} \frac{(l - m)!}{(n + m)!} \int_B \left(\frac{r}{r_o}\right)^l P_{lm}(\sin \delta) CS(m\lambda) \sigma dV \quad (4.10)$$

$$\sigma dV = \sigma(r, \delta, \lambda) r^2 \cos \delta dr d\delta d\lambda \quad (4.11)$$

where CS stands for the cosine or sine function for calculating the C_{lm} or S_{lm} gravity coefficient, respectively, and r_o is an arbitrary reference radius that is frequently chosen to equal the maximum or mean radius of the body. The gravity coefficients would otherwise have length to the l th power as units of length l th; however, the reference radius r_o guarantees that the coefficients are dimensionless numbers. One integration over the radius can be carried out for a body with a constant density, and the expression becomes:

$$(C, S)_{lm} = \frac{\sigma(2 - \delta_m^0)}{M} \frac{(l - m)!}{(n + m)!} \int_s \frac{R^3(\delta, \lambda)}{l + 3} \left(\frac{R(\delta, \lambda)}{r_o}\right)^l P_{lm}(\sin \delta) CS(m\lambda) \cos \delta d\delta d\lambda \quad (4.12)$$

Hence the body's shape is defined by the radius $R(\delta, \lambda)$, which is now a function of latitude and longitude [3].

Many studies have been conducted in the fields of geodesy and astrodynamics about the impact of Earth's non-spherical form on Earth satellites as a result of gravitational potential. There are deviations from a totally spherical shape in the Earth's gravitational field because of differences in mass distribution. Mathematical representations of the Earth's gravitational field called zonal harmonics which are mathematical representations of the Earth's gravitational field [16].

Zonal Harmonics

are determined by zeroth order ($l = 0$), at which point the domain is symmetric about the axis of Polarity and potential's dependency on longitude disappears. These are merely latitude bands. There exist ^{1} latitude circles along which P_l is zero for every $P_l[\sin(\phi_{gsat})]$, and hence, $(+1)$ zones in which the function alternately increases (shaded) and decreases (white). Because of the curvature of the strongest perturbation on Earth is definitely J_2 . Multiple representations of the zonal harmonics are shown in Figure 2 [14].

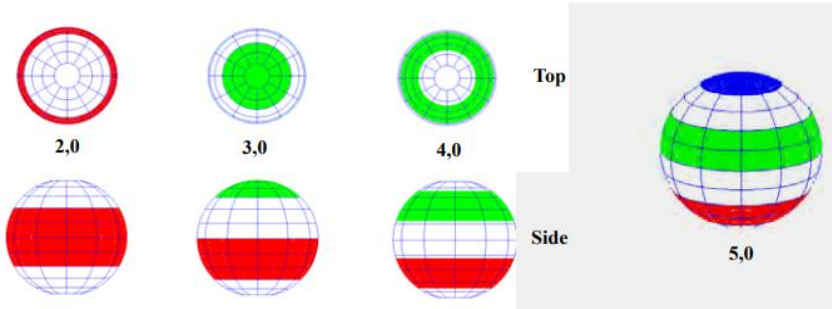


Figure 2: Zonal Harmonics. [8, 15].

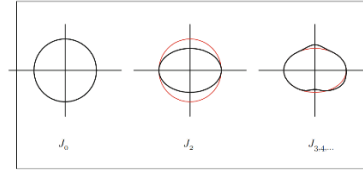


Figure 3: Changing views on Earth's shape in geodesy [3].

Sectoral Harmonics

occur when ($l = n$) and serve as longitude bands. A few sectoral harmonics are displayed in Figure 4. $P_{l,l}$ and $[\text{SIN}(\phi_{gcsat})]$ Polynomials are zero at just the poles ($\phi_{gsat} = \mp 90^\circ$). Conversely, given two distinct values of λ , the terms $[\text{COS}(\lambda)]$ and $[\text{SIN}(\lambda)]$ are zero. Therefore, the lines that disappear along the functions $[\text{COS}(\lambda)]$ and $[\text{SIN}(\lambda)]$ times $P_{l,l}$ and $[\text{SIN}(\phi_{gcsat})]$ are meridians of longitude, dividing the sphere into two sectors known as "orange slices." Positive (shaded) and negative (white) mass concentrations are represented by each sector [14, 18].

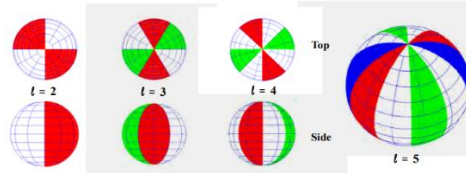


Figure 4: Sectoral Harmonics. [8, 15].

Tesseral Harmonics

(as they resemble tiles), when the spherelit into a checkerboard pattern. While the terms $[\text{cos}(m\lambda)]$ and $[\text{SIN}(m\lambda)]$ disappear along $2m$ meridians of longitude, the number of circles of latitude along which $P_{l,m}[\text{SIN}(\phi_{gcsat})]$ is zero is equal to $(l - m)$. The latitude and longitude bands' centers are represented by these zero lines. For the tesseral harmonics, see Figure 5. Observe the parallels and divergences with the sectoral harmonics. The sectorals are referred to in some literature as a subset of the tesserals. That's true, but we can also keep them apart so that we can tell apart their physical characteristics [8, 18].

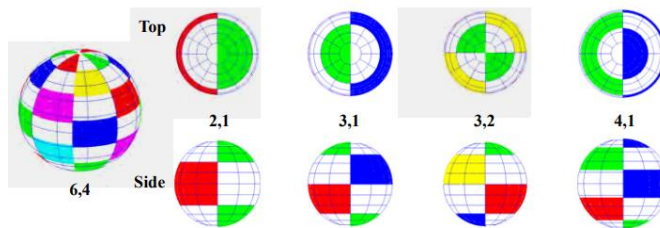


Figure 5: Tesseral Harmonics [14, 18].

FINDINGS AND DISCUSSION

We aim to clarify in this study how Earth satellites in the non-spherical Earth are

affected by the gravitational potential, which may be expressed as a set of spherical harmonic functions. These spherical harmonics characterize the deviation of the real pear-shaped earth from the two-body problem's spherically symmetric earth. Because the earth's density can vary (be it higher or lower) along a given latitude, the zonal harmonics explain how the earth's actual shape deviates from the symmetrical Kepler Earth in terms of latitude. For this reason, the physical effects resulting from the zonal harmonics are crucial when tracking satellites in space. Utilizing the SATORB program [2], A test satellite's orbit is integrated over a comparatively short period of time—one day. The study presents numerical output results from a software program for celestial mechanics, including orbital elements, coordinate systems, Kepler's orbits, and acceleration components for various satellites and altitudes. The data entered for the orbital elements is given in Table 1.

Table 1: Input Data.

	Period(min)	h (km)	a(km)	e	i (deg)	Ω (deg)	ω (deg)
LOW orbit	91.535	350	6728	0.006614	40	112.2	4.6
	96.871	608	6987	0.006614	40	112.2	4.6
	102.116	859	7237	0.006614	40	112.2	4.6
Medium orbit	225.437	5891	12270	0.006614	40	112.2	4.6
	287.932	8065	14444	0.006614	40	112.2	4.6
	360.148	10390	16768	0.006614	40	112.2	4.6
High orbit	645.69	19129	25507	0.006614	40	112.2	4.6
	861.568	23615	29993	0.006614	40	112.2	4.6
	1436.009	35785	42163	0.006614	40	112.2	4.6

we set the degree ($N=50$) and attempt to ascertain the effects of the zonal harmonics. These harmonics characterize the latitude deviations between the symmetrical Kepler earth and the real earth's form; since the earth's density can vary at varying latitude lines (higher or lower), the physical consequences resulting from the zonal harmonics are of great importance when tracking satellites. Figure 6 illustrates how perturbations affect the orbital elements for three groups of satellites at three distinct altitudes (low, medium, and high). Over one day, the osculating orbital elements are extracted and supplied as a function of the Modified Julian Date, the figures show that, in general, the overwhelming influence on the elements (semi. Major Axes, eccentricity, and inclination) is periodic, but the other three elements exhibit secular variation in addition to brief periodic variations.

Table 2: Represents the Effect of Perturbation (Zonal $N = 50$) for One Day.

Type of perturbation	h (km)	a(km)	e	i (deg)	Ω (deg)	ω (deg)	M(deg)
		periodic	periodic	periodic	secular	secular	secular
Low orbit	350	8.4	0.0022	0.041	-6.3	14.5	13
	608	8	0.002	0.039	-5.5	11.5	11
	859	7.8	0.00185	0.034	-4.8	10.5	9.5
Medium orbit	5891	4	0.00063	0.0121	0.8	3.2	3.1
	8065	3.6	0.00045	0.0080	0.42	2.3	2.2
	10390	2.9	0.00032	0.0065	0.24	1.6	1.7
High orbit	19129	1.7	0.00014	0.0028	0.06	0.6	0.75
	23615	1.6	0.0001	0.002	0.035	0.5	0.55
	35785	0.9	0.000055	0.001	0.01	0.3	0.22

Periodic variations in the ascending node, the perigee argument, and the mean anomaly, the primary long-term consequences of the non-spherical earth disturbance are these secular variations. As a result of the torque created by the equator's bulge, the plane of the orbit of the satellite tends to turn in the direction of the equator. This

phenomenon, known as the regression of the node, occurs when the aspherical earth's satellite crosses the equator before the unperturbed, two-body satellite does. As altitude (the semi-major axis) rises, the secular and periodic fluctuations' magnitudes diminish. Table 2 indicates the computation and structure of the disturbance effect on the orbital elements, the results indicate that the influence of the disturbance decreases with increasing satellite height.

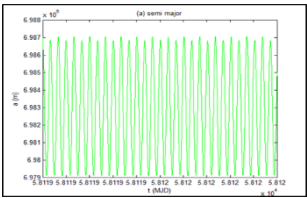
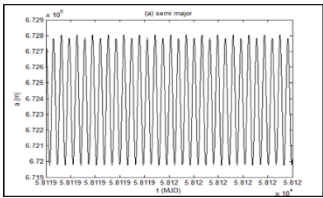
The changes concerning the first osculating Keplerian orbit (in radial-along track-and out of plane directions, Zonal harmonic degree 50 (one day) can be measured in meters in the low orbit and medium orbit, shown in Figure 8. The respect to the first Keplerian orbit that is osculating (in radial, along-track, and out-of-plane directions) for high orbit and argument of latitude for (low, medium, high) orbit in Zonal harmonic degree 50 (one day). displayed in Figure 9, The short periodic variations in the along-track direction coexist with a secular variation, with the periodic effect primarily impacting the components that are radial. We observe cyclical growth out of the plane because of the zonal harmonic coefficient., The largest deviations from the Keplerian orbits caused by the non-spherical disturbance of the central body are indicated

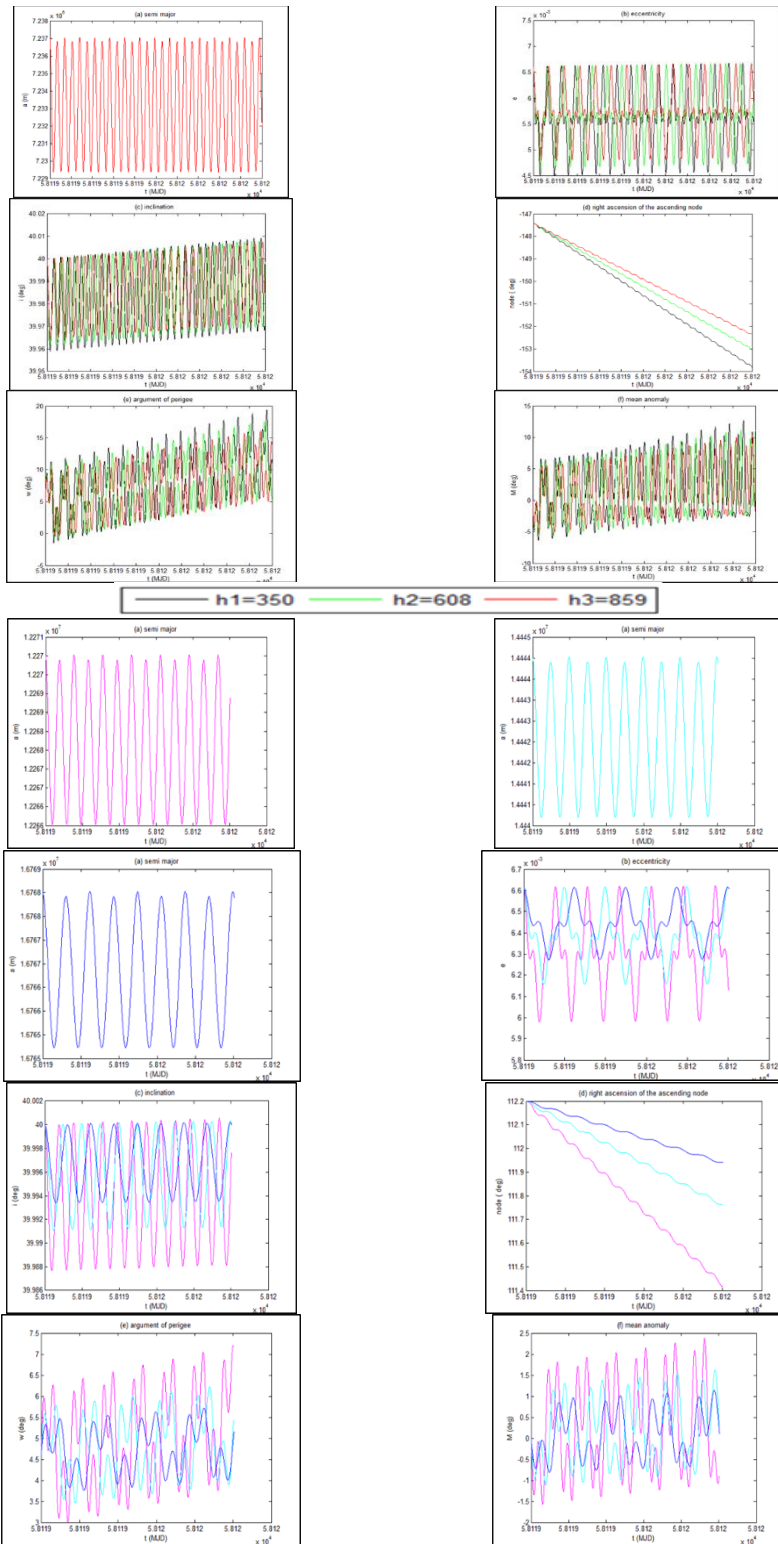
Table 3: Represents the Effect of Keplerian Orbit (In Radial, Along-Track, and Out-Of-Plane Directions).

Type of orbit	h(km)	Short period One Day		
		Radial (km)	Cross-track (km)	Along-track (km)
Low orbit	350	36	900	5800
		608	840	5100
		859	770	4800
Medium orbit	5891	8.8	180	3000
		8065	120	2300
		10390	100	2000
High orbit	19129	3.8	28	780
		23615	17	580
		35785	07	350

CONCLUSION

The Earth's non-sphericity due to the non-homogeneous distribution of mass causes a different gravitational potential to affect the satellites orbiting around it. Therefore, in this research, the extent to which the orbital elements of the satellites are affected when adopting the value of the factor N=50 was studied. Based on the findings, we may conclude that the effects of gravitational potential on the orbital elements decrease as the altitude of the satellite increases. The study also included the extent to which the differences about the original Keplerian osculating orbit may be found (in radial, along-track, and out-of-plane directions) osculating Keplerian compounds are affected by changes in the satellite's altitude. It was noted from the results that the satellite's elevation has an important role in the extent of its influence on gravitational potential.





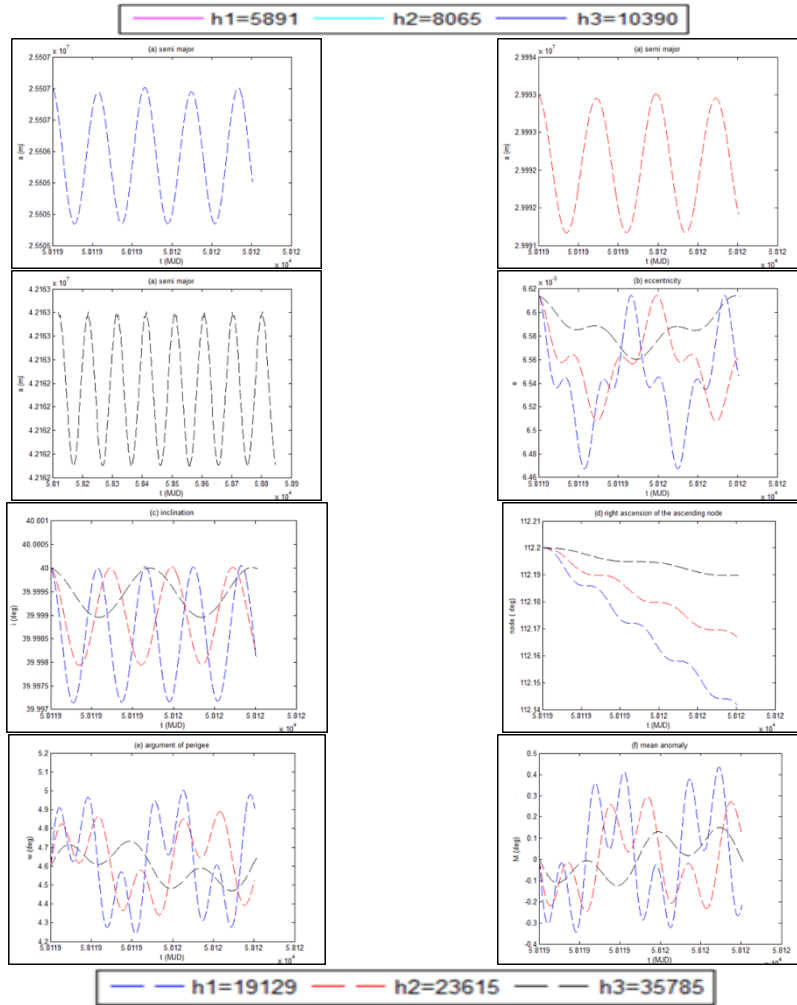
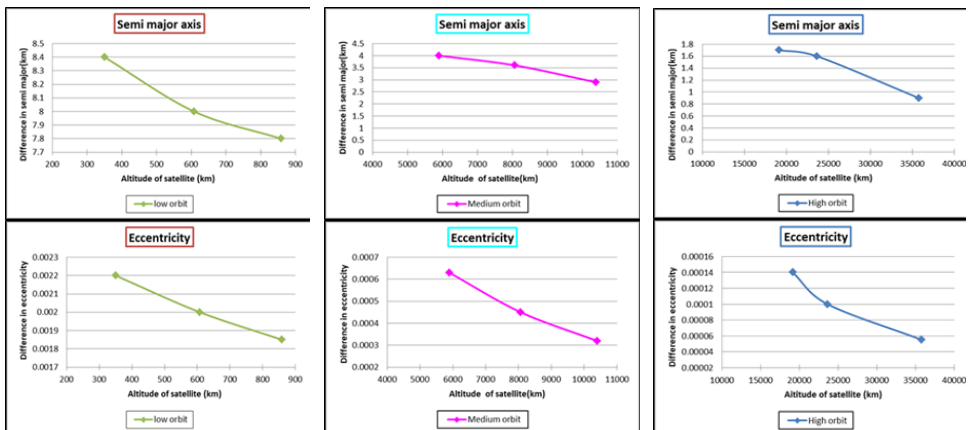


Figure 6: Perturbations Effect on Orbital Elements of Satellites at Different Altitudes.



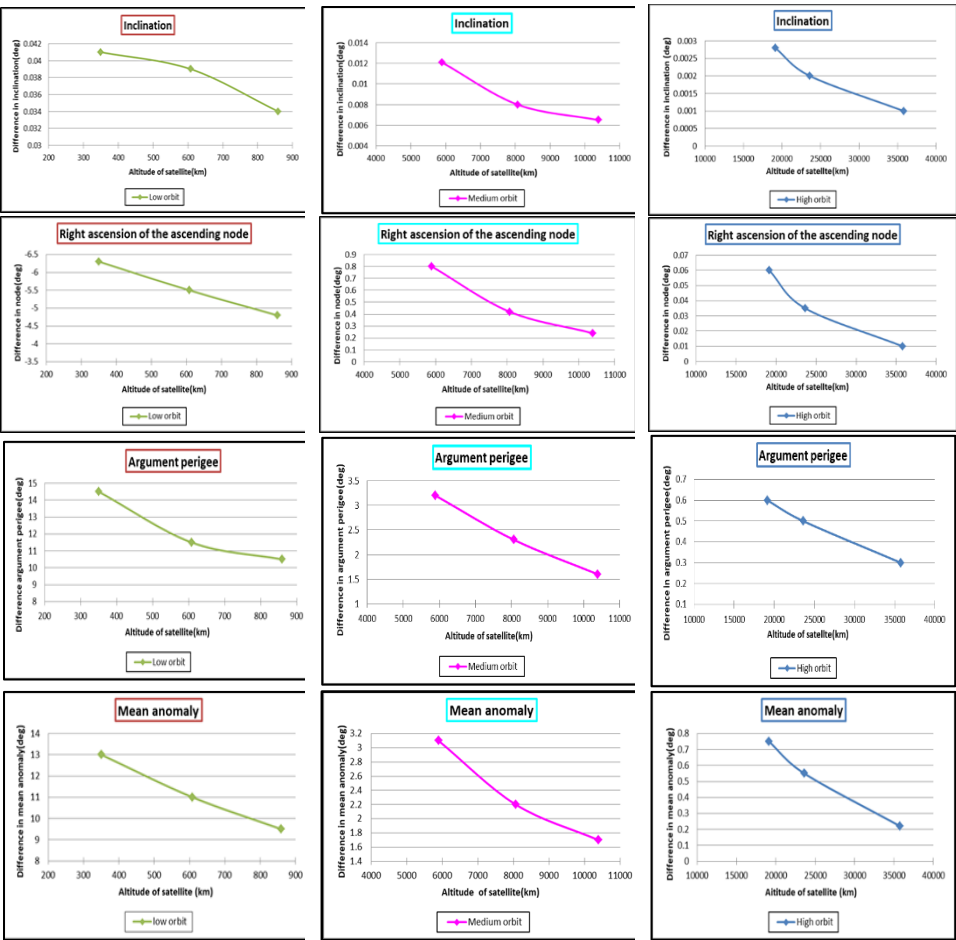
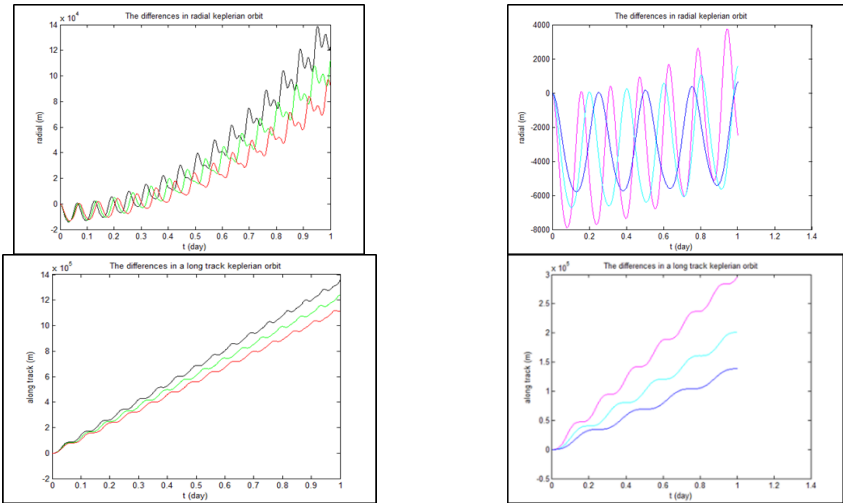


Figure 7: Represents the Deviation in Orbital Elements Due to Non-Spherical Earth Perturbation for One Day.



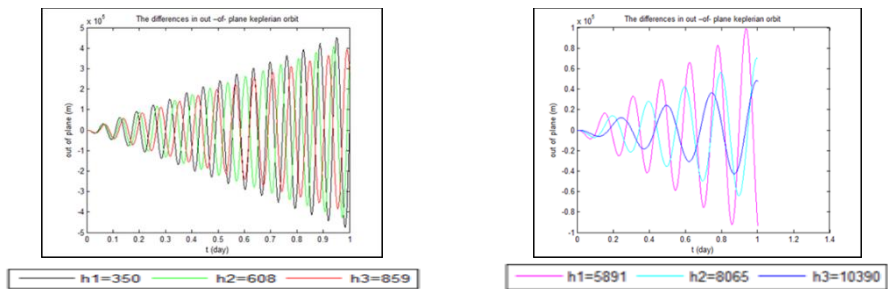


Figure 8: Change In Relation to the First Osculating Keplerian Orbit for Low and Medium Orbit.

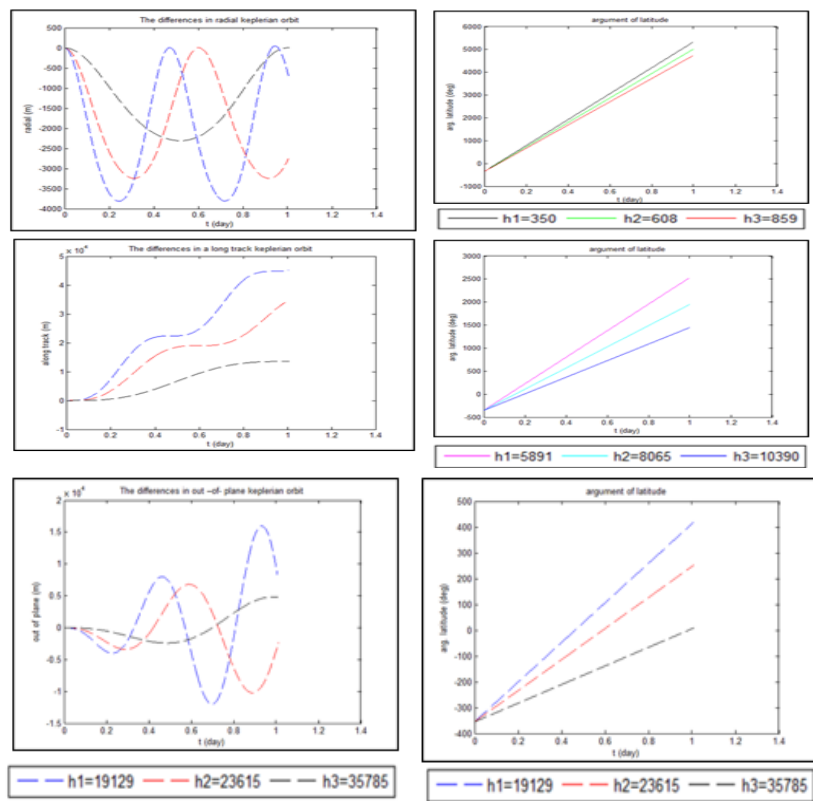
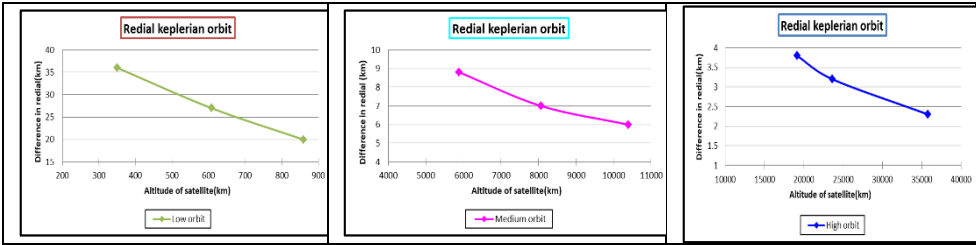


Figure 9: Changes In Relation to The First Osculating Keplerian Orbit for High Orbit.



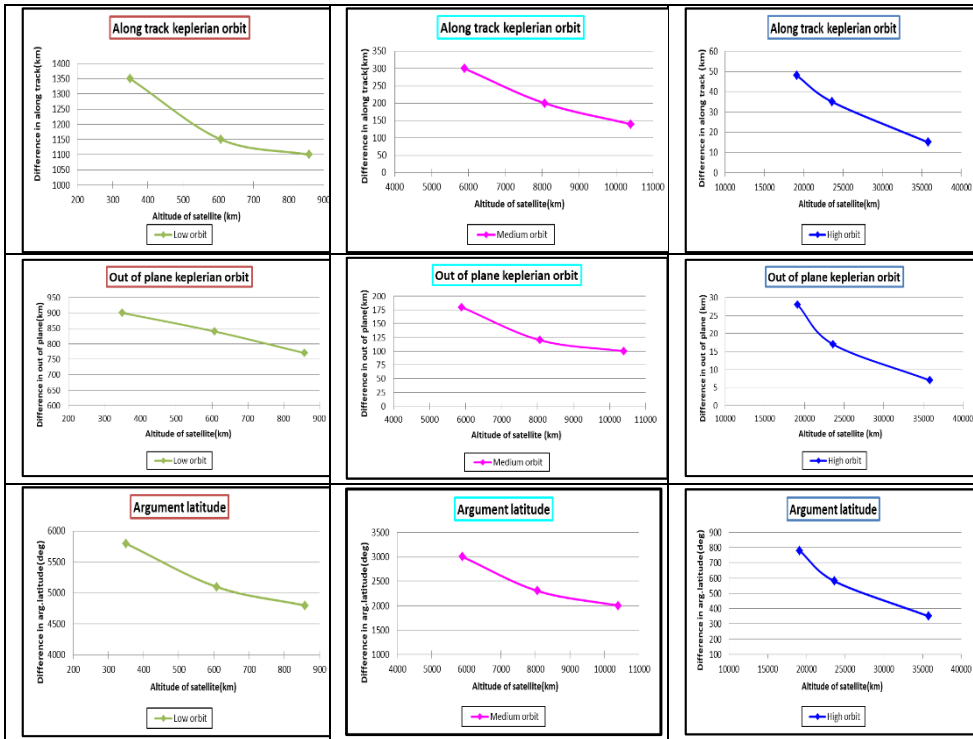


Figure 10: Represent the Deviation in Osculating Keplerian Compounds due to Non-spherical Earth.

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